UNIT 1. SIGNALS AND SYSTEM

INTRODUCTION

A SIGNAL is defined as any physical quantity that changes with time, distance, speed, position, pressure, temperature or some other quantity. A SIGNAL is physical quantity that consists of many sinusoidal of different amplitudes and frequencies.

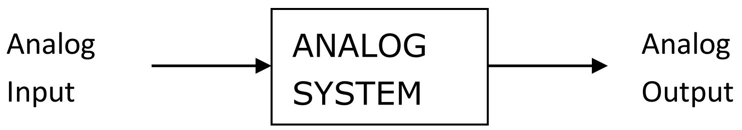
Ex $x(t) = 10t$

$X(t) = 5x^2+20xy+30y$

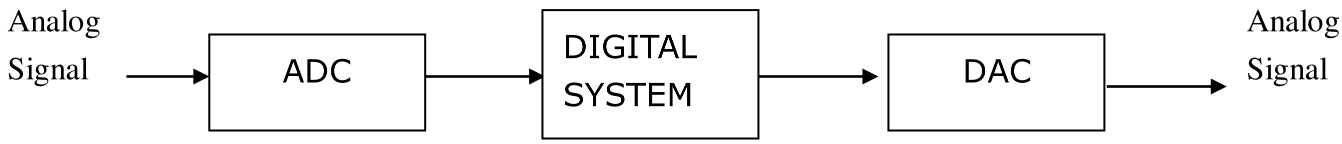
A System is a physical device that performs an operations or processing on a signal. Ex Filter or Amplifier.

CLASSIFICATION OF SIGNAL PROCESSING

1) ASP (Analog signal Processing): If the input signal given to the system is analog then system does analog signal processing. Example: Resistor, capacitor or Inductor, OP-AMP etc.



2) DSP (Digital signal Processing): If the input signal given to the system is digital then system does digital signal processing. Example: Digital Computer, Digital Logic Circuits etc. The devices called as ADC (analog to digital Converter) converts Analog signal into digital and DAC (Digital to Analog Converter) does vice-versa.



Most of the signals generated are analog in nature. Hence these signals are converted to digital form by the analog to digital converter. Thus AD Converter generates an array of samples and gives it to the digital signal processor. This array of samples or sequence of samples is the digital equivalent of input analog signal. The DSP performs signal processing operations like filtering, multiplication, transformation or amplification etc operations over this digital signal. The digital output signal from the DSP is given to the DAC.

ADVANTAGES OF DSP OVER ASP

1. Physical size of analog systems is quite large while digital processors are more compact and light in weight.
2. Analog systems are less accurate because of component tolerance ex R, L, C and active components. Digital components are less sensitive to the environmental changes, noise and disturbances.
3. Digital systems are most flexible as software programs & control programs can be easily modified.
4. Digital signal can be stores on digital hard disk, floppy disk or magnetic tapes. Hence becomes transportable. Thus easy and lasting storage capacity.
5. Digital processing can be done offline.
6. Mathematical signal processing algorithm can be routinely implemented on digital signal processing systems. Digital controllers are capable of performing complex computation with constant accuracy at high speed.
7. Digital signal processing systems are upgradeable since that are software controlled.
8. Possibility of sharing DSP processor between several tasks.

9. The cost of microprocessors, controllers and DSP processors are continuously going down. For some complex control functions, it is not practically feasible to construct analog controllers.
10. Single chip microprocessors, controllers and DSP processors are more versatile and powerful.

Disadvantages of DSP over ASP

1. Additional complexity (A/D & D/A Converters)
2. Limit in frequency. High speed AD converters are difficult to achieve in practice. In high frequency applications DSP are not preferred.

CLASSIFICATION OF SIGNALS

1. Single channel and Multi-channel signals
2. Single dimensional and Multi-dimensional signals
3. Continuous time and Discrete time signals.
4. Continuous valued and discrete valued signals.
5. Analog and digital signals.
6. Deterministic and Random signals
7. Periodic signal and Non-periodic signal
8. Symmetrical(even) and Anti-Symmetrical(odd) signal
9. Energy and Power signal

1) Single channel and Multi-channel signals

If signal is generated from single sensor or source it is called as single channel signal. If the signals are generated from multiple sensors or multiple sources or multiple signals are generated from same source called as Multi-channel signal. Example ECG signals. Multi-channel signal will be the vector sum of signals generated from multiple sources.

2) Single Dimensional (1-D) and Multi-Dimensional signals (M-D)

If signal is a function of one independent variable it is called as single dimensional signal like speech signal and if signal is function of M independent variables called as Multi-dimensional signals. Gray scale level of image or Intensity at particular pixel on black and white TV is examples of M-D signals.

3) Continuous time and Discrete time signals.

Sr No	Continuous Time (CTS)	Discrete time (DTS)
1	This signal can be defined at any time instance & they can take all values in the continuous interval(a, b) where a can be -∞ & b can be ∞	This signal can be defined only at certain specific values of time. These time instance need not be equidistant but in practice they are usually takes at equally spaced intervals.
2	These are described by differential equations.	These are described by difference equation.
3	This signal is denoted by x(t).	These signals are denoted by x(n) or notation x(nT) can also be used.
4	The speed control of a dc motor using a techo-generator feedback or Sine or exponential waveforms.	Microprocessors and computer based systems uses discrete time signals.

3) Continuous valued and Discrete Valued signals.

Sr. No	Continuous Valued	Discrete Valued
1	If a signal takes on all possible values on a finite or infinite range, it is said to be continuous valued signal.	If signal takes values from a finite set of possible values, it is said to be discrete valued signal.
2	Continuous Valued and continuous time signals are basically analog signals.	Discrete time signal with set of discrete amplitude are called digital signal.

5) Analog and digital signal

Sr. No	Analog signal	Digital signal
1	These are basically continuous time & continuous amplitude signals.	These are basically discrete time signals & discrete amplitude signals. These signals are basically obtained by sampling & quantization process.
2	ECG signals, Speech signal, Television signal etc. All the signals generated from various sources in nature are analog.	All signal representation in computers and digital signal processors are digital.

Digital signals (DISCRETE TIME & DISCRETE AMPLITUDE) are obtained by sampling the ANALOG signal at discrete instants of time, obtaining DISCRETE TIME signals and then by quantizing its values to a set of discrete values & thus generating DISCRETE AMPLITUDE signals.

Sampling process takes place on x axis at regular intervals & quantization process takes place along y axis. Quantization process is also called as rounding or truncating or approximation process.

6) Deterministic and Random signals

Sr. No	Deterministic signals	Random signals
1	Deterministic signals can be represented or described by a mathematical equation or lookup table.	Random signals that cannot be represented or described by a mathematical equation or lookup table.
2	Deterministic signals are preferable because for analysis and processing of signals we can use mathematical model of the signal.	Not Preferable. The random signals can be described with the help of their statistical properties.
3	The value of the deterministic signal can be evaluated at time (past, present or future) without certainty.	The value of the random signal cannot be evaluated at any instant of time.
4	Example Sine or exponential waveforms.	Example Noise signal or Speech signal

7) Periodic signal and Non-Periodic signal

The signal x(n) is said to be periodic if x(n+N)= x(n) for all n where N is the fundamental period of the signal. If the signal does not satisfy above property called as Non-Periodic signals.

Discrete time signal is periodic if its frequency can be expressed as a ratio of two integers. f= k/N where k is integer constant.

9) Energy signal and Power signal

Discrete time signals are also classified as finite energy or finite average power signals.

The energy of a discrete time signal x(n) is given by

$$E=\sum_{n=-\infty}^{\infty}|x^2(n)|$$

The average power for a discrete time signal x (n) is defined as

If Energy is finite and power is zero for x(n) then x(n) is an energy signal. If power is finite and energy is infinite then x(n) is power signal. There are few signals which are neither energy nor a power signals.

<u>**DISCRETE TIME SIGNALS AND SYSTEM**</u>

There are three ways to represent discrete time signals.

1) **Functional Representation**

$$x(n)=\begin{cases} 4 & \text{for } n=1, 3 \\ -2 & \text{for } n=2 \\ 0 & \text{elsewhere} \end{cases}$$

2) **Tabular method of representation**

n	-3	-2	-1	0	1	2	3	4	5
x(n)	0	0	0	0	4	-2	4	0	0

3) **Sequence Representation**

$$X(n) = \{0, 4, \underset{\substack{\uparrow \\ n=0}}{-2}, 4, 0, \ldots\ldots\}$$

<u>**STANDARD SIGNAL SEQUENCES**</u>

1) **Unit sample signal (Unit impulse signal)**

$$\delta(n) = \begin{cases} 1 & n=0 \\ 0 & n=0 \end{cases} \qquad \text{i.e. } \delta(n)=\{1\}$$

2) **Unit step signal**

$$u(n) = \begin{cases} 1 & n\geq 0 \\ 0 & n<0 \end{cases}$$

3) **Unit ramp signal**

$$u(n) = \begin{cases} n & n\geq 0 \\ 0 & n<0 \end{cases}$$

4) **Exponential signal**

$$X(n) = a^n = (re^{j\emptyset})^n = r^n e^{j\emptyset n} = r^n(\cos \emptyset n + j \sin \emptyset n)$$

5) **Sinusoidal waveform**

X (n) = A Sin wn

OPERATIONS ON DISCRETE TIME SIGNALS

1) **Shifting:** signal x (n) can be shifted in time. We can delay the sequence or advance the sequence. This is done by replacing integer n by n-k where k is integer. If k is positive signal is delayed in time by k samples (Arrow get shifted on left hand side) and if k is negative signal is advanced in time k samples (Arrow get shifted on right hand side)

X(n) = { 1, -1 , 0 , 4 , -2 , 4 , 0 ,……}

n=0

Delayed by 2 samples : X(n-2)= { 1, -1 , 0 , 4 , -2 , 4 , 0 ,……}

n=0

Advanced by 2 samples : X(n+2) = { 1, -1 , 0 , 4 , -2 , 4 , 0 ,……}

n=0

2) **Folding / Reflection**: It is folding of signal about time origin n=0. In this case replace n by –n.

Original signal:

X (n) = { 1, -1 , 0 , 4 , -2 , 4 , 0}

n=0

Folded signal:

X (-n) = { 0 , 4 , -2 , 4 , 0 , -1 , 1}

n=0

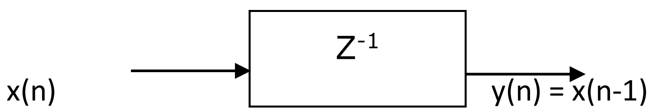
3) **Addition**: Given signals are x1(n) and x2(n), which produces output y(n) where y(n) = x1(n)+ x2(n). Adder generates the output sequence which is the sum of input sequences.

4) **Scaling**: Amplitude scaling can be done by multiplying signal with some constant. Suppose original signal is x(n). Then output signal is A x (n)

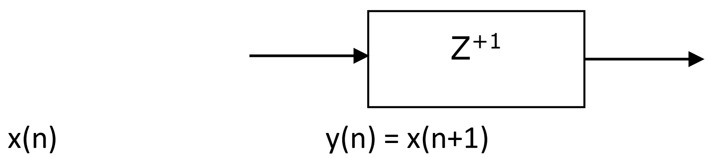
5) **Multiplication**: The product of two signals is defined as y(n) = x1(n) * x2(n).

SYMBOLS USED IN DISCRETE TIME SYSTEM

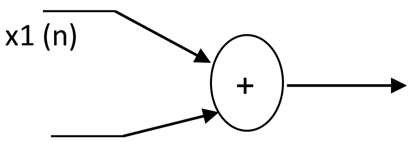
1. Unit delay



2. Unit advance

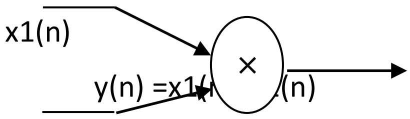


3. Addition



x2 (n)   y (n) =x1(n)+x2(n)

4. Multiplication



x2 (n)

5. Scaling (constant multiplier)

$$X(n) \xrightarrow{A} \quad y(n) = A\,x(n)$$

CLASSIFICATION OF DISCRETE TIME SYSTEMS

1) STATIC v/s DYNAMIC

Sr. No	STATIC	DYNAMIC (Dynamicity property)
1	Static systems are those systems whose output at any instance of time depends at most on input sample at same time.	Dynamic systems output depends upon past or future samples of input.
2	Static systems are memory less systems.	They have memories for memorize all samples.

It is very easy to find out that given system is static or dynamic. Just check that output of the system solely depends upon present input only, not dependent upon past or future.

Sr. No	System [y(n)]	Static / Dynamic	Remark
1	x(n)	Static	As System depends on present value only
2	x(n-2)	Dynamic	As System depends on past value hence needs memory for storage
3	$x^2(n)$	Static	As System depends on present value only
4	$x(n^2)$	Dynamic	As System depends on future values also
5	$n\,x(n) + x^2(n)$	Static	As System depends on present value only
6	x(n)+ x(n-2) +x(n+2)	Dynamic	As System depends on past and future values also

2) TIME INVARIANT v/s TIME VARIANT SYSTEMS

Sr No	TIME INVARIANT (TIV) / SHIFT INVARIANT	TIME VARIANT SYSTEMS / SHIFT VARIANT SYSTEMS (Shift Invariance property)
1	A System is time invariant if its input output characteristic does not change with shift of time.	A System is time variant if its input output characteristic changes with time.
2	Linear TIV systems can be uniquely characterized by Impulse response, frequency response or transfer function.	No Mathematical analysis can be performed.
3	a. Thermal Noise in Electronic components b. Printing documents by a printer	a. Rainfall per month b. Noise Effect

It is very easy to find out that given system is Shift Invariant or Shift Variant.

Suppose if the system produces output y(n) by taking input x(n)

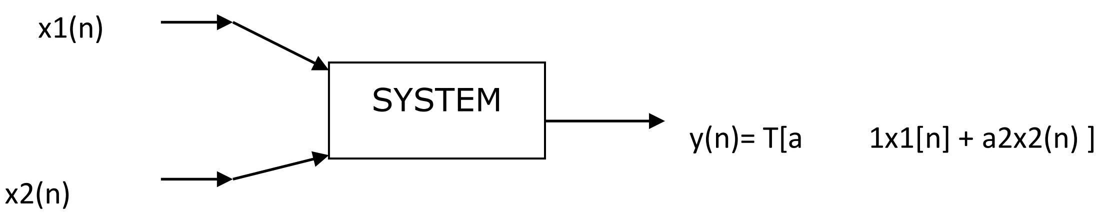
x(n) → y(n)

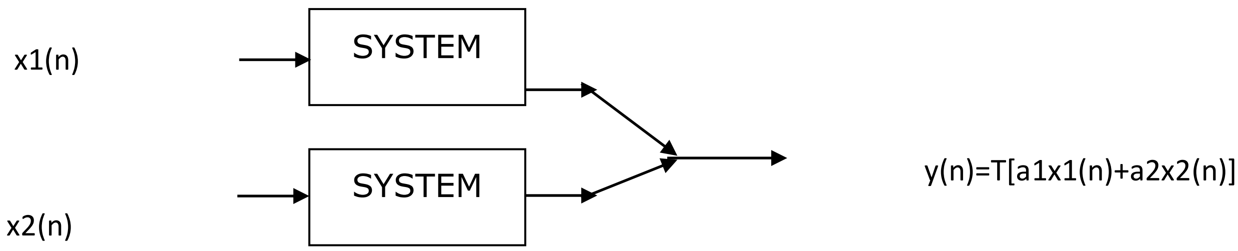
If we delay same input by k units x(n-k) and apply it to same systems, the system produces output y(n-k)

$$x(n-k) \rightarrow y(n-k)$$

3) LINEAR v/s NON-LINEAR SYSTEMS

Sr. No	LINEAR	NON-LINEAR (Linearity Property)
1	A System is linear if it satisfies superposition theorem.	A System is Non-linear if it does not satisfy superposition theorem.
2	Let x1(n) and x2(n) are two input sequences, then the system is said to be linear if and only if T[a1x1(n) + a2x2(n)]=a1T[x1(n)]+a2T[x2(n)]	





Hence T [a1 x1(n) + a2 x2(n)] = T [a1 x1(n)] + T [a2 x2(n)]

It is very easy to find out that given system is linear or Non-Linear.

Response to the system to the sum of signal = sum of individual responses of the system.

Sr. No	System y(n)	Linear or Non-Linear	Remark
1	$e^{x(n)}$	Non-Linear	System does not satisfy superposition theorem.
2	x^2 (n)	Non-Linear	System does not satisfy superposition theorem.
3	cos [x(n)]	Non-Linear	System does not satisfy superposition theorem.
4	X(-n)	Linear	System does not satisfy superposition theorem.
5	Log_{10} (|x(n)|)	Non-Linear	System does not satisfy superposition theorem.

4) CAUSAL v/s NON CAUSAL SYSTEMS

Sr. No	CAUSAL	NON-CAUSAL (Causality Property)
1	A System is causal if output of system at any time depend only past and present inputs.	A System is Non causal if output of system at any time depends on future inputs.
2	In Causal systems the output is the function of x(n), x(n-1), x(n-2)….. and so on.	In Non-Causal System the output is the function of future inputs also. X(n+1) x(n+2) .. and so on
3	Example Real time DSP Systems	Offline Systems

It is very easy to find out that given system is causal or non-causal. Just check that output of the system depends upon present or past inputs only, not dependent upon future.

Sr. No	System [y(n)]	Causal /Non-Causal	Remark
1	x(n) + x(n-3)	Causal	Output of system depends on past and present values
2	X(n)	Causal	Output of system does not depend future value
3	X(n) + x(n+3)	Non-Causal	Output of system at any time depends on future inputs.
4	2 x(n)	Causal	Output of system does not depend future value
5	X(2n)	Non-Causal	Output of system depend future value
6	X(n)+ x(n-2) +x(n+2)	Non-Causal	Output of system depend future value

5) STABLE v/s UNSTABLE SYSTEMS

Sr. No	STABLE	UNSTABLE (Stability Property)
1	A System is BIBO stable if every bounded input produces a bounded output.	A System is unstable if any bounded input produces a unbounded output.
2	The input x(n) is said to bounded if there exists some finite number M_x such that $\|x(n)\| \le M_x < \infty$ The output y(n) is said to bounded if there exists some finite number M_y such that $\|y(n)\| \le M_y < \infty$	

STABILITY FOR LTI SYSTEM

It is very easy to find out that given system is stable or unstable. Just check that by providing input signal check that output should not rise to ∞.

The condition for stability is given by

$$\sum_{k=-\infty}^{\infty} |h(k)| < \infty$$

Sr No	System [y(n)]	Stable / Unstable	Remark
1	Cos [x(n)]	Stable	Cosine function produces bounded output
2	x(-n+2)	Stable	System produces bounded output
3	|x(n)|	Stable	System produces bounded output
4	x(n) u(n)	Stable	System produces bounded output
5	X(n) + n x(n+1)	Unstable	System produces unbounded output

ANALYSIS OF DISCRETE LINEAR TIME INVARIANT (LTI/LSI) SYSTEM

1) CONVOLUTION SUM METHOD

2) DIFFERENCE EQUATION

LINEAR CONVOLUTION SUM METHOD

1. This method is powerful analysis tool for studying LSI Systems.
2. In this method we decompose input signal into sum of elementary signal. Now the elementary input signals are taken into account and individually given to the system. Now using linearity property whatever output response

we get for decomposed input signal, we simply add it & this will provide us total response of the system to any given input signal.

3. Convolution involves folding, shifting, multiplication and summation operations.

4. If there are M number of samples in x(n) and N number of samples in h(n) then the maximum number of samples in y(n) is equals to M+n-1.

Linear Convolution states that

$$y(n) = x(n) * h(n)$$

$$y(n) = \sum_{k=-\infty}^{\infty} x(k)h(n-k)$$

Example 1: h(n) = { 1 , $\underline{2}$, 1, -1 } & x(n) = { $\underline{1}$, 2, 3, 1 } Find y(n)

METHOD 1: GRAPHICAL REPRESENTATION

Step 1) Find the value of n = n_x+ n_h = -1 (Starting Index of x(n)+ starting index of h(n))

Step 2) y(n)= { y(-1) , y(0) , y(1), y(2), ….} It goes up to length x(n)+ length y(n) -1.

i.e. n=-1 y(-1) = x(k) * h(-1-k)

n=0 y(0) = x(k) * h(0-k)

n=1 y(1) = x(k) * h(1-k) ….

ANSWER : y(n) ={1, 4, 8, 8, 3, -2, -1 }

METHOD 2: MATHEMATICAL FORMULA

Use Convolution formula

$$y(n) = \sum_{k=-\infty}^{\infty} x(k)h(n-k)$$

k= 0 to 3 (start index to end index of x (n))

y(n) = x(0) h(n) + x(1) h(n-1) + x(2) h(n-2) + x(3) h(n-3)

METHOD 3: VECTOR FORM (TABULATION METHOD)

X(n)= {x_1,x_2,x_3} & h(n) ={ h_1,h_2,h_3}

	x1	x2	x3
h	h_1x1	h_1x2	h_1x3
h	h_2x1	h_2x2	h_2x3
h	h_3x1	h_3x2	h_3x3

y(-1) = h_1 x_1

y(0) = h_2 x_1 + h_1 x_2

y(1) = h_1 x_3 + $h_2$$x_2$ + h_3 x_1 …………

PROPERTIES OF LINEAR CONVOLUTION

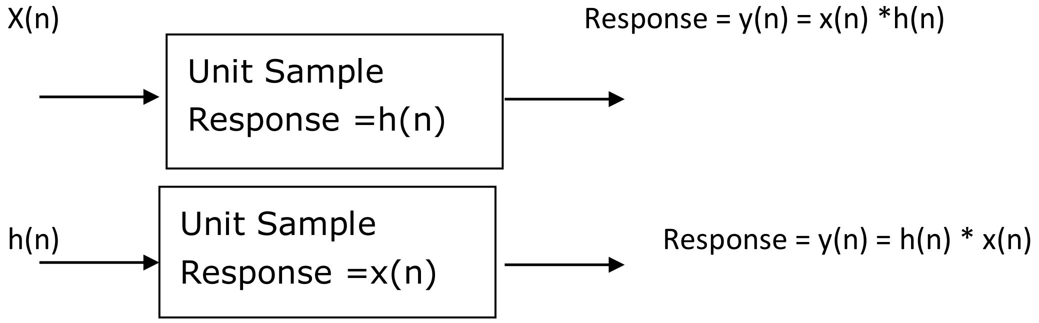
x(n) = Excitation Input signal

y(n) = Output Response

h(n) = Unit sample response

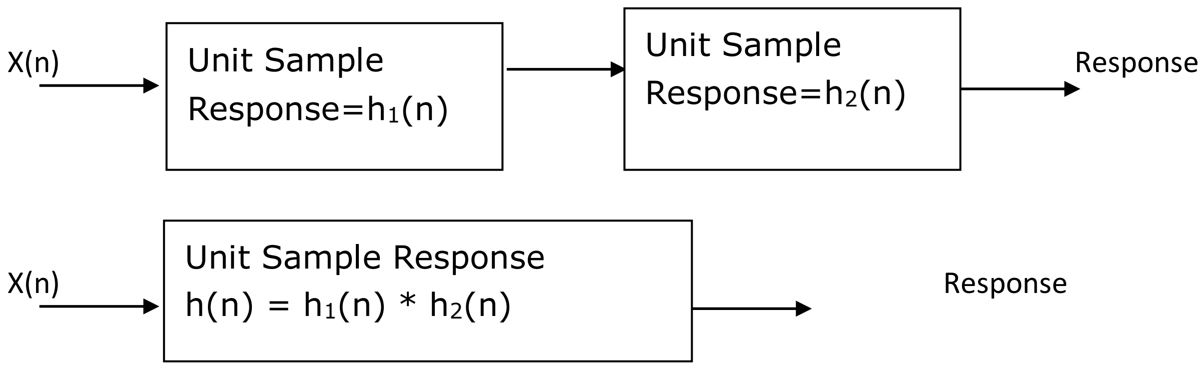
1. **Commutative Law: (Commutative Property of Convolution)**

$x(n) * h(n) = h(n) * x(n)$



2. **Associate Law: (Associative Property of Convolution)**

$[x(n) * h_1(n)] * h_2(n) = x(n) * [h_1(n) * h_2(n)]$



3 **Distribute Law: (Distributive property of convolution)**

$x(n) * [h_1(n) + h_2(n)] = x(n) * h_1(n) + x(n) * h_2(n)$

CAUSALITY OF LSI SYSTEM

The output of causal system depends upon the present and past inputs. The output of the causal system at n= n_0 depends only upon inputs x (n) for n≤ n_0. The linear convolution is given as

$$y(n)= \sum_{k=-\infty}^{\infty} h(k)x(n-k)$$

At n= n_0 ,the output $y(n_0)$ will be

$$y(n_0)= \sum_{k=-\infty}^{\infty} h(k)x(n_0-k)$$

The output of causal system at n= n_0 depends upon the inputs for n< n_0 Hence

h (-1)=h(-2)=h(-3)=0

Thus LSI system is causal if and only if

h (n) =0 for n<0

This is the necessary and sufficient condition for causality of the system.

Linear convolution of the causal LSI system is given by

$$y(n)= \sum_{k=0}^{n} x(k)h(n-k)$$

STABILITY FOR LSI SYSTEM

A System is said to be stable if every bounded input produces a bounded output.

The input x(n) is said to bounded if there exists some finite number M_x such that $|x(n)| \le M_x < \infty$. The output y(n) is said to bounded if there exists some finite number M_y such that $|y(n)| \le M_y < \infty$.

Linear convolution is given by

$$y(n)= \sum_{k=-\infty}^{\infty} x(k)h(n-k)$$

Taking the absolute value of both sides

$$|y(n)|=|\sum_{k=-\infty}^{\infty} x(k)h(n-k)|$$

The absolute values of total sum is always less than or equal to sum of the absolute values of individually terms. Hence

$$|y(n)| \leq |\sum_{k=-\infty}^{\infty} x(k)h(n-k)|$$

$$|y(n)| \leq \sum_{k=-\infty}^{\infty} |x(k)||h(n-k)|$$

The input x(n) is said to bounded if there exists some finite number M_x such that $|x(n)| \leq M_x < \infty$. Hence bounded input x(n) produces bounded output y(n) in the LSI system only if

$$\sum_{k=-\infty}^{\infty} |h(k)| < \infty$$

With this condition satisfied, the system will be stable. The above equation states that the LSI system is stable if its unit sample response is absolutely sum able. This is necessary and sufficient condition for the stability of LSI system.

DIFFERENCE EQUATION

Sr. No	**Finite Impulse Response (FIR)**	**Infinite Impulse Response (IIR)**
1	FIR has an impulse response that is zero outside of some finite time interval.	IIR has an impulse response on infinite time interval.
2	Convolution formula changes to $y(n)= \sum_{n=-M}^{M} x(k)h(n-k)$ For causal FIR systems limits changes to 0 to M.	Convolution formula changes to $y(n)= \sum_{n=-\infty}^{\infty} x(k)h(n-k)$ For causal IIR systems limits changes to 0 to ∞.
3	The FIR system has limited span which views only most recent M input signal samples forming output called as "Windowing".	The IIR system has unlimited span.
4	FIR has limited or finite memory requirements.	IIR System requires infinite memory.
5	Realization of FIR system is generally based on Convolution Sum Method.	Realization of IIR system is generally based on Difference Method.

Discrete time systems have one more type of classification.

1. Recursive Systems
2. Non-Recursive Systems

Sr. No	**Recursive Systems**	**Non-Recursive systems**
1	In Recursive systems, the output depends upon past, present, future value of inputs as well as past output.	In Non-Recursive systems, the output depends only upon past, present or future values of inputs.
2	Recursive Systems has feedback from output to input.	No Feedback.
3	Examples y(n) = x(n) + y(n-2)	Y(n) = x(n) + x(n-1)

First order Difference Equation

y(n) = x(n) + a y(n-1)

Where y (n) = Output Response of the recursive system

X (n) = Input signal

a= Scaling factor

y (n-1) = Unit delay to output.

Now we will start at n=0

n=0 y (0) = x(0) + a y(-1) ....(1)

n=1 y (1) = x(1) + a y(0) ....(2)

= x (1) + a [x (0) + a y(-1)]

= a^2 y (-1) + a x (0) + x (1) ....(3)

Hence

Y(n)=a^{n+1} y(-1)+$\sum_{k=0}^{n} a^k$x(n-k) $n \geq 0$

1) The first part (A) is response depending upon initial condition.
2) The second Part (B) is the response of the system to an input signal.

Zero state response (Forced response) : Consider initial condition are zero. (System is relaxed at time n=0)

i.e. $y(-1) = 0$

Zero Input response (Natural response) : No input is forced as system is in non-relaxed initial condition. i.e.

$y(-1) \neq 0$

Total response is the sum of zero state response and zero input response.

Q.1 A Discrete time system is represented by the following equation

y(n)=$\frac{3}{2}$y(n-1)-$\frac{1}{2}$y(n-2)+x(n)

With initial conditions

y(-1)=0

y(-2)=-2

and x(n)=$(\frac{1}{4})^n$

Determine

(i)Zero input response

(ii) Zero state response

(iii) Total response of the system

Sol. The given equation can be written as

y(n)-$\frac{3}{2}$y(n-1)+$\frac{1}{2}$y(n-2)=x(n)

let us obtain the characteristic equation with zero input

y(n)-$\frac{3}{2}$y(n-1)+$\frac{1}{2}$y(n-2)=0 ---(1)

Assume the solution

$$y_h(n) = \lambda^n$$

$\lambda^n - \frac{3}{2}\lambda^{n-1} + \frac{1}{2}\lambda^{n-2}$=0

$\lambda^{n-2}(\lambda^2 - \frac{3}{2} + \frac{1}{2})$=0

$$\lambda^2 - \frac{3}{2}\lambda + \frac{1}{2} = 0$$

Multiplying by 2

$$2\lambda^2 - 3\lambda + 1 = 0$$
$$2\lambda^2 - 2\lambda - \lambda + 1 = 0$$
$$2\lambda(\lambda - 1) - 1(\lambda - 1) = 0$$

λ=1,$\frac{1}{2}$

Hence

$y_h(n) = C_1(1)^n$+$C_2(\frac{1}{2})^n$ ----(2)

For finding the values of C_1 and C_2
Putting n=0 in (1)

$1 - \frac{3}{2}$=$C_2/2$

y(0)= $\frac{3}{2}$y(-1) -$\frac{1}{2}$y(-2) =0+1 --(3)

Putting n=1 in (1)

y(1)= $\frac{3}{2}$y(0) -$\frac{1}{2}$y(-1) =$\frac{3}{2}$ --(4)

Putting n=0 in (2)
$y(0) = C_1$+C_2 --(5)
y(1)= C_1+$C_2/2$ --(6)

(5)-(6)
y(0)-y(1)= $C_2/2$

-$\frac{1}{2}$=$C_2/2$

$$C_2 = -1$$

$C_1 = y(0)$ -C_2 =$1 - (-1)$=2

So $y_h(n) = 2(1)^n - (\frac{1}{2})^n$ $y_h(n) = 2 - (\frac{1}{2})^n$ ----(7)

This is the required zero input response
ii) To determine the zero state response (forced response)
Assume the particular solution

$y_p(n)$=$k(\frac{1}{4})^n$u(n) ---(8)

From (1) and (8)

$k(\frac{1}{4})^n$u(n)- $k\frac{3}{2}(\frac{1}{4})^{n-1}$u(n-1)+ $k\frac{1}{2}(\frac{1}{4})^{n-2}$u(n-2)= $(\frac{1}{4})^n$u(n)

For n≥ 2

Above equation can be written as

$\left(\frac{1}{4}\right)^{n-2}(\frac{1}{16}k - k\frac{3}{8} + k\frac{1}{2} - \frac{1}{16})$=0

$$\frac{1}{16}k - k\frac{3}{8} + k\frac{1}{2} - \frac{1}{16} = 0$$

Multiplying by 16

k-6k+8k-1=0

3k=1

k=$\frac{1}{3}$

Hence $y_p(n)$=$\frac{1}{3}(\frac{1}{4})^n$u(n) -----(9)

Forced response

$y_n(n)$=$y_h(n)$+$y_p(n)$ -----(10)

$y_n(n)$=$y_h(n) + y_p(n)$

$= C_1$+$C_2(\frac{1}{2})^n$ +$\frac{1}{3}(\frac{1}{4})^n$ ------(11)

Putting n=0 in(1)

y(0)= $\frac{3}{2}$y(-1) -$\frac{1}{2}$y(-2)+x(0)

for zero state response initial conditions are zero, hence

y(-1)=y(-2)=0

x(0)= $(\frac{1}{4})^0$=1

therefore y(0)=1 ----(14)

with n=1

y(1)= $\frac{3}{2}$y(0) -$\frac{1}{2}$y(-1)+x(1) =$\frac{3}{2}$+$(\frac{1}{4})^1$=$\frac{3}{2}$+$\frac{1}{4}$

=$\frac{7}{4}$ ------(15)

Putting n=0 in (11)

$y(0) = C_1$+$C_2 + \frac{1}{3}$ ---(12)

Putting n=1 in (11)

$y(1) = C_1$+$\frac{1}{2}C_2 + \frac{1}{3}$ *$\frac{1}{4}$

$y(1) = C_1$+$\frac{1}{2}C_2 + \frac{1}{12}$ ----(13)

Eq. (12)-(13)

y(0)-y(1)=$\frac{1}{2}C_2$+$\frac{1}{4}$ ---(14)

y(0)-y(1)= $\frac{-3}{4}$ ------(15)

from (14) and (15)

$\frac{1}{2}C_2+\frac{1}{4} = \frac{-3}{4}$

$\frac{1}{2}C_2$=-1

C_2=-2 --- (16)

From Eq. (12),(14) and (16)

$$1 = C_1 - 2 + \frac{1}{3}$$

C_1=3-$\frac{1}{3}$=$\frac{8}{3}$ ---(17)

From (11), (16) and (17)

Forced response is

$$= \frac{8}{3} - 2(\frac{1}{2})^n + \frac{1}{3}(\frac{1}{4})^n$$

And Total response is summation of natural response and forced response

$$y(n) = 2 - (\frac{1}{2})^n + \frac{8}{3} - 2(\frac{1}{2})^n + \frac{1}{3}(\frac{1}{4})^n$$

$=\frac{14}{3}-3(\frac{1}{2})^n+\frac{1}{3}(\frac{1}{4})^n$

Q.2 Determine the range of values 'a' and 'b' for which the LTI system with impulse response

$h(n)=\begin{cases} a^n, n \geq 0 \\ b^n \ , n < 0 \end{cases}$

is stable .

Solution

The condition of stability is

$$\sum_{k=-\infty}^{\infty} |h(k)| < \infty$$

Splitting the summation and substituting to the given equation

$\sum_{n=-\infty}^{\infty}|h(n)| = \sum_{n=0}^{\infty}|a^n| + \sum_{n=-\infty}^{-1}|b^n|$

The first summation can be written as

$\sum_{n=0}^{\infty}|a^n|$ =1+$|a|$+$|a^2|$+$|a^3|$+……

This is the standard geometric series and converges to $\frac{1}{1-|a|}$ if $|a|$ <1. If $|a|$ >1, the series does not converge and it becomes unstable. Thus

$\sum_{n=0}^{\infty}|a^n| = \frac{1}{1-|a|}$ if $|a|$ <1

By considering the second summation

$\sum_{n=-\infty}^{-1}|b^n| = \sum_{n=-\infty}^{-1}\left|\frac{1}{b^n}\right|$

$= \frac{1}{|b|} + \frac{1}{|b^2|} + \frac{1}{|b^3|} + \ldots$

$\frac{1}{|b|}\left[1 + \frac{1}{|b|} + \frac{1}{|b^2|} + \frac{1}{|b^3|} + \ldots\right]$

The part inside the brackets, is the geometric series and it converges to $\frac{1}{1-\frac{1}{|b|}}$ if $\frac{1}{|b|} < 1$ i.e. $|b| > 1$

$\sum_{n=-\infty}^{-1}|b^n| = \frac{1}{|b|} \cdot \frac{1}{1-\frac{1}{|b|}}$ if $|b| > 1$

Substituting the values of summations obtained in the above equation

$\sum_{n=-\infty}^{\infty}|h(n)| = \frac{1}{1-|a|} + \frac{1}{|b|-1}$ if $|a| < 1 > |b|$

Thus the geometric series converges if $|a| < 1 > |b|$ Or system will be stable if $|a| < 1 > |b|$.