Unit 1

Syllabus: Radiation

Potential function and the Electromagnetic field, potential functions for Sinusoidal Oscillations, retarded potential, the Alternating current element (or oscillating Electric Dipole), Power radiated by a current element, Application to short antennas, Assumed current distribution, Radiation from a Quarter wave monopole or Half wave dipole, sine and cosine integral, Electromagnetic field close to an antenna, Solution of the potential equations, Far-field Approximation.

1.1 Introduction:

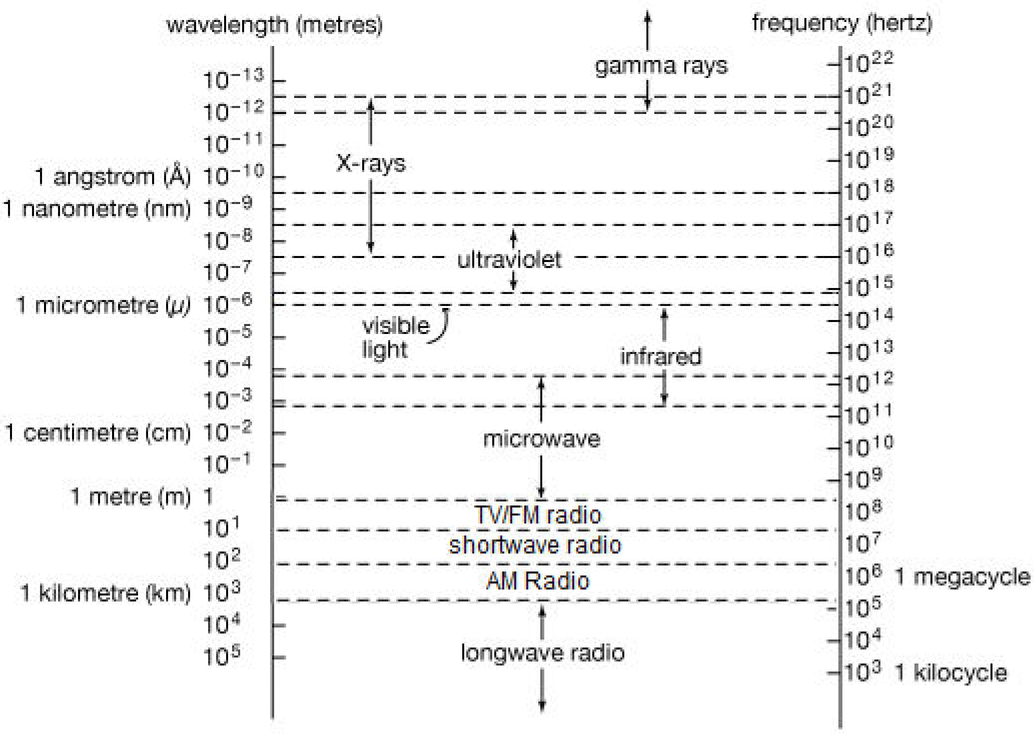
Antenna may be considered as a metallic device for radiating or receiving radio waves. It is a transitional structure between free space and a guiding device i.e. transmission line.

The various commonly used antennas are whip antennas on cars, single turn loop antennas for UHF TV receiver, roof mounted log-periodic antenna and satellite parabolic reflector receiving antennas. These commonly used antennas represent only a small segment of antenna systems that were being developed for specialized and high performance communication links like RADAR system, navigation system and scientific study.

Wave Propagation: The performance of a communication links depends not only on the antenna used but also it is strongly influenced by atmosphere and conductivity of ground.

Electromagnetic spectrum and frequency band designation: The electromagnetic wave energy radiated by antenna oscillates at radio frequency. The wavelength of a wave is related to frequency f and velocity c of a wave by

$$C = f\lambda$$



The classification of the radio waves, their nomenclature modes and typical services are shown in table 1.01. The range of frequencies is broken down into several bands designated as shown in table 1.02.

Table 1.01

S. No.	Frequency	Designation	Typical Service
01	3 – 30 Hz	Extremely Low Frequency (ELF)	Detection of buried metal objects
02	30 – 300 Hz	Super Low Frequency (SLF)	Ionospheric Sensing, Electric Power Distribution, Submarine Communication
03	300 Hz – 3KHz	Ultra Low Frequency (ULF)	Audio Signals on Telephone
04	3 – 30 KHz	Very Low Frequency (VLF)	Navigation Sonar, position location
05	30 – 300 KHz	Low Frequency (LF)	Radio Beacons, Weather broadcast stations for air navigation
06	300 – 3000 KHz	Medium Frequency (MF)	AM Broadcasting, Coast Guard Communication, direction finding
07	3 – 30 MHz	High Frequency (HF)	Telephone, Telegraph, short wave broadcasting
08	30 – 300 MHz	Very High Frequency (VHF)	Television, FM Broadcast, Air Traffic Control, Police, Mobile Radio Communication
09	300 – 3000 MHz	Ultra High Frequency (UHF)	Television, Satellite Communication, RADAR, Microwave Ovens, Cellular Telephone
10	3 – 30 GHz	Super High Frequency (SHF)	Airborne RADAR, Microwave links, Satellite Comm., Remote Sensing, radio astronomy
11	30 – 300 GHz	Extremely High Frequency (EHF)	RADAR, Advanced Communication Systems, Remote Sensing, radio astronomy

Table 1.02

S. No.	Frequency	Microwave Band Designation	
		Old	New
01	500 – 1000 MHz	VHF	C
02	1 – 2 GHz	L	D
03	2 – 3 GHz	S	E
04	3 – 4 GHz	S	F
05	4 – 6 GHz	C	G
06	6 – 8 GHz	C	H
07	8 – 10 GHz	X	I
08	10 – 12.4 GHz	X	J
09	12.4 – 18 GHz	Ku	J
10	18 – 20 GHz	K	J
11	20 – 26.5 GHz	K	K
12	26.5 – 40 GHz	Ka	K

1.2 Review of Electromagnetic Theory

Electromagnetic fields are produced by time-varying charge distributions which can be supported by time-varying current distributions. Consider sinusoidally varying electromagnetic sources. Sources having arbitrary variation with respect to time can be represented in terms of sinusoidally varying functions using Fourier analysis. A sinusoidally varying current i(t) can be expressed as a function of time, t, as

$$i(t) = I_0 Cos(\omega t + \varphi)$$

where I_0 is the amplitude ,ω is the angular frequency, and φ is the phase. The angular frequency, ω, is related to the frequency, f by the relation ω = 2πf. The current *i(t)* is given by

$$i(t) = I_0 Sin(\omega t + \varphi)$$

Where φ' = φ + π/2. Therefore, we need to identify whether the phase has been defined taking the cosine function or the sine function as a reference. We have chosen the cosine function as the reference to define the phase of the sinusoidal quantity.
Since $cos(\omega t + \phi)=Re\{e^{j(\omega t+\phi)}\}$ where, *Re{}* represents the real part of the quantity within the curly brackets, the current can now be written as

$$i(t) = I_0 \, Re\{e^{j(\omega t+\varphi)}\}$$
$$= Re\{I_0 e^{j\varphi} e^{j\omega t}\}$$

The quantity $I_0 e^{j\varphi}$ is known as a phasor and contains the amplitude and phase information of *i(t)* but is independent of time, t.

1.3 Fundamental of Electromagnetic Radiation

An antenna is a structure usually made of good conducting material, designed to have a shape and size such that it radiates electromagnetic power in an efficient manner. When antenna is excited by time varying currents, it radiates electromagnetic waves. In order to radiate efficiently, the minimum antenna size must be comparable to the wavelength.
To calculate the field radiated by an antenna Maxwell's equations and an auxiliary function, Retarded Potential is used. Maxwell's equations describe all electromagnetic phenomena.

$$\nabla \times E = -\partial B / \partial t \qquad ...1.3.1$$

$$\nabla \times H = J + \frac{\partial D}{\partial t} \qquad ...1.3.2$$

(Maxwell's Equations)

$$\nabla . \mathrm{D} = \rho \qquad ...1.3.3$$

$$\nabla . \mathrm{B} = 0 \qquad ...1.3.4$$

Where E : Electric field intensity (unit: volt per metre, V/m)
H : Magnetic field intensity (unit: ampere per metre, A/m)
D : Electric flux density (unit: coulomb per metre, C/m)
B : Magnetic flux density (unit: weber per metre, Wb/m or tesla, T)
J : Current density (unit: ampere per square metre, A/m2)
ρ : Charge density (unit: coulomb per cubic metre, C/m3)

The first is Faraday's law of induction; the second is Ampere's law as amended by Maxwell to include the displacement current ∂D/∂t, the third and fourth are Gauss's laws for the electric and magnetic fields. The displacement current term ∂D/∂t in Ampere's law is essential in predicting the existence of propagating electromagnetic waves. Eqs. (1.1.1) are in SI units. The quantities E and H are the electric and magnetic field intensities and are measured in units of [volt/m] and [ampere/m], respectively. The quantities D and B are the electric and magnetic flux densities and are in units of [coulomb/m2] and [Weber/m2], or [tesla]. B is also called the magnetic induction.

The quantities ρ and J are the volume charge density and electric current density of any external charges. They are measured in units of [coulomb/m3] and [ampere/m2]. The right-hand side of the fourth equation is zero because there are no magnetic monopole charges.
The charge and current densities ρ, J may be thought of as the sources of the electromagnetic fields. For wave propagation problems, these densities are localized in space; for example, they are restricted to flow on an antenna. The generated electric and magnetic fields are radiated away from these sources and can propagate to large distances to the receiving antennas. Away from the sources, that is, in source-free regions of space, Maxwell's equations take the simpler form:

$$\nabla \times E = -\partial B / \partial t \quad \text{...1.3.5}$$

$$\nabla \times H = J + \frac{\partial D}{\partial t} \quad \text{...1.3.6}$$

(Source Free Maxwell's Equations)

$$\nabla . \mathrm{D} = 0 \quad \text{...1.3.7}$$

$$\nabla . \mathrm{B} = 0 \quad \text{...1.3.8}$$

1.4 Basic Maxwell's Equations

Maxwell's equations can be written in differential and integral forms. For the present study, the differential form of equations is more suited. The relevant equations involving electric field intensity E, electric flux density D, magnetic field intensity H, magnetic flux density B, current density J and the charge density ρ are as given below.

$\nabla \times H = J + \partial D/\partial t$ (in general),

$\nabla \times H = \partial D/\partial t$ (if J = 0) and $\nabla \times H = J$ (For DC Field) ...1.4.1

$\nabla \times E = -\partial B/\partial t$ (in general) and $\nabla \times E = 0$ (For static Field) ...1.4.2

$\nabla . \mathrm{D} = \rho$ (in general) and $\nabla . \mathrm{D} = 0$ (for charge free region i.e., $\rho = 0$) ...1.4.3

$\nabla . \mathrm{B} = 0$ ...1.4.4

The field quantities involved in equations 1.4.1 and 1.4.3 are connected by the following relations:

$D = \varepsilon E$ ...1.4.5

$B = \mu H$ ...1.4.6

$J = \sigma E = E/\rho$ ...1.4.7

here ε is the permittivity, μ is the permeability, σ is the conductivity and ρ is the resistivity *(ρ = 1/σ)* of the media. It is to be noted that the symbol ρ represents entirely different quantities.
Besides the above, the other relevant relations are

$$V = \int \frac{\rho l \partial l}{4\pi\varepsilon R} = \int\int \frac{\rho s \partial s}{4\pi\varepsilon R} = \int\int\int \frac{\rho v \partial v}{4\pi\varepsilon R} \quad \text{...1.4.8}$$

$$E = -\nabla V \quad \text{...1.4.9}$$

$\nabla^2 V = \frac{-\rho}{E}$ (in general) and $\nabla^2 V = 0$ if $\rho=0$ ...1.4.10

In equation 1.4.8, V is the scalar electric potential; ρl , ρs and ρv are line, surface and volume charge densities; and R is the distance between the source and the point at which V is to be evaluated.

$$A = \int \frac{\mu I \partial l}{4\pi R} = \int\int \frac{\mu K \partial s}{4\pi R} = \int\int\int \frac{\mu J \partial v}{4\pi R} \quad ...1.4.11$$

$$B = \nabla x A \quad ...1.4.12$$

$\nabla^2 A = -\mu J$ (in general) and $\nabla^2 A = 0$ if $J=0$ ...1.4.13

In 1.4.11, A is the vector magnetic potential, I is the current, K is the surface current density and J and R are the same as defined earlier.

1.5 Retarded (Time Varying) Potential

Some of the relations listed above are derived for the static or dc field conditions. Since radiation is a time varying phenomena, the validity of these relations needs to be tested. To start with consider Eq. 1.4.9 When its curl is taken, it is noted that

$$\nabla \times E = \nabla \times (-\nabla V) \equiv 0 \quad ...1.5.1$$

This result is obtained in view of the vector identity that the curl of a gradient is identically zero.

But from 1.4.2, $\nabla \times E = -\partial B/\partial t$ for a time-varying field.

The discrepancy is obvious and can be addressed by using 1.4.12.
Let

$$E = -\nabla V + N \quad ...1.5.2$$

$$\nabla \times E = \nabla \times (-\nabla V) + \nabla \times N = 0 + \nabla \times N = -\partial B/\partial t = -\partial(\nabla \times A)/\partial t$$

Thus, $\nabla \times N = -\partial(\nabla \times A)/\partial t = -\nabla \times \partial A/\partial t = \nabla \times (-\partial A/\partial t)$

Or, $\nabla \times N = -\partial A/\partial t$ ...1.5.3

Substitution of equation 1.5.3 in section 1.3 gives a new relation 1.5.4 which satisfies both the static and the time-varying conditions:

$$E = -\nabla V - \partial A/\partial t \quad ...1.5.4$$

In the second step, the validity of third equation of (1.4.3) is to be tested by using the relation of (1.4.5) and 1.5.4

$$\begin{aligned} \nabla . D &= \nabla .(\varepsilon E) = \varepsilon \nabla . E \\ &= \varepsilon \nabla .(-\nabla V - \partial A/\partial t) \\ &= \varepsilon(-\nabla . \nabla V - \partial/\partial t(\nabla . A)) = \rho \end{aligned}$$

From the above relation,

$$\nabla^2 V + \partial/\partial t(\nabla . A)) = -\rho/\varepsilon$$ ...1.5.5

The RHS of 1.5.4 leads to the following relations:

$$\nabla^2 V = -\rho/\varepsilon$$ for static conditions ...1.5.6

$$\nabla^2 V = -\rho/\varepsilon - \partial/\partial t(\nabla . A)$$ for time-varying conditions ...1.5.7

In the third step, the validity of 1.4.1 is to be tested by using the relations of equations 1.4.6, 1.4.12 and 1.5.4

$$\nabla \times H = J + \partial D/\partial t$$ ...1.5.8

$$B = \mu H \ or\ H = B/\mu$$

The LHS of 1.5.8 can be written as

$$LHS = (\nabla \times B)/\mu = (\nabla \times \nabla \times A)/\mu = [\nabla(\nabla.\mathrm{A}) - \nabla^2 A]/\mu$$ ...1.5.9

This relation uses the vector identity $\nabla \times \nabla \times A \equiv \nabla(\nabla.\mathrm{A}) - \nabla^2 A$ ...1.5.10

The RHS of 1.5.8 can also be written as

$$RHS = J + \varepsilon\, \partial D/\partial t = J + \varepsilon\, \partial(-\nabla V - \partial A/\partial t)/\partial t$$

$$= J + \varepsilon[-\nabla(\partial V/\partial t) - \partial^2 A/\partial t^2]$$

$$= J - \varepsilon[\nabla(\partial V/\partial t) + \partial^2 A/\partial t^2]$$ ...1.5.11

On equating LHS and RHS terms, we get

$$\nabla(\nabla.\mathrm{A}) - \nabla^2 A = \mu J - \mu\varepsilon[\nabla(\partial V/\partial t) + \partial^2 A/\partial t^2$$ ...1.5.12

In 1.5.7 and 1.5.12, the term ∇2A is defined in (1.21), whereas the term ∇ ·A is yet to be defined. As per the statement of Helmholtz Theorem, “A vector field is completely defined only when both its curl and divergence are known”. There are some conditions which specify divergence of A. Two of these conditions, known as Lorentz gauge condition and Coulomb’s gauge condition, are given by 1.5.11 and 1.5.11 respectively.

$$\nabla.\mathrm{A} = -\mu\varepsilon\partial V/\partial t$$ ...1.5.13

$$\nabla.\mathrm{A} = 0$$ ...1.5.14

Using the Lorentz gauge condition, 1.5.7 and 1.5.12 can be rewritten as

$$\nabla^2 V = -\rho/\varepsilon - \partial(\mu\varepsilon\partial V/\partial t)/\partial t = -\rho/\varepsilon - \mu\varepsilon(\partial^2 V/\partial t^2)$$ ...1.5.15

$$\nabla^2 V = -\mu J + \mu\varepsilon(\partial^2 A/\partial t^2)$$ ...1.5.16

For sinusoidal time variation characterized by $e^{j\omega t}$

$V = V_0 e^{j\omega t}$ and $A = A_0 e^{j\omega t}$

$$\nabla^2 V = -\rho/\varepsilon + \omega^2 \mu\varepsilon V$$ ...1.5.17

$$\nabla^2 A = -\mu J + \omega^2 \mu \varepsilon J \quad \text{...1.5.18}$$

If ρ and J in the expressions of V and A given by 1.4.8 and 1.4.11 of Sec 1.3, they become functions of time and this time t is replaced by t' such that *t' = t-r/v. ρ* and *J* can be replaced by [ρ] and [J] respectively. Equation 1.4.8 and 1.4.11 of Sec 1.3 can now be rewritten as

$$V = \int_v \frac{[\rho]dv}{4\pi\varepsilon R} \quad \text{...1.5.19}$$

$$A = \int_v \frac{\mu[J]dv}{4\pi R} \quad \text{...1.5.20}$$

As an example if $\rho = e^{-r} \cos \omega t$, and t is replaced by t', one gets $[\rho] = e^{-r} \cos[\omega(t - R/\upsilon)]$. In this expression, R is the distance between the elemental volume dv located in a current-carrying conductor and the point P as shown in Fig. 4–1, and υ is the velocity with which the field progresses or the wave travels. V and A given by 1.5.19 and 1.5.20 are called the retarded potentials. If t'_ = t + r/υ, V and A are termed as advanced potentials.

With Reference to the figure 1.1, equations 1.5.19 and 1.5.20 can be written as

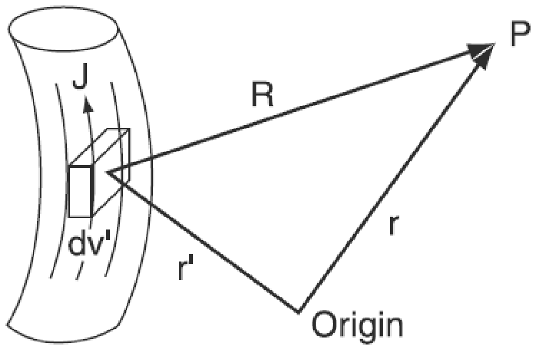


Figure 1-1 Geometry of the configuration containing the element volume dv and an arbitrary point P.

$$V(r,t) = \frac{1}{4\pi\varepsilon} \int_v \frac{\rho(r', t)}{R} dv' \quad \text{...1.5.21}$$

$$A(r,t) = \frac{\mu}{4\pi} \int_v \frac{J(r', t)}{R} dv' \quad \text{...1.5.22}$$

In 1.5.19 and 1.5.20,V and A are the functions of the distance r and the time t. To get the retarded potentials from (19) and (20), t is to be replaced by t' and the resulting field equations are

$$V(r,t) = \frac{1}{4\pi\varepsilon} \int_v \frac{\rho(r', t - R/v)}{R} dv' \quad \text{...1.5.23}$$

$$A(r,t) = \frac{\mu}{4\pi} \int_v \frac{J(r', t - R/v)}{R} dv' \quad \text{...1.5.24}$$

Similarly, advanced potential expression can be obtained by replacing t −R/υ by t +R/υ in 1.5.21 and 1.5.22. Equation 1.5.22, the starting point for the study of radiation process, is rewritten in the following alternating form on replacing R by r.

$$A(r) = \frac{\mu}{4\pi} \int_v \frac{J(t - r/v)}{r} dv \quad \text{...1.5.25}$$

1.6 Far Field due to an Alternating Current Element (Oscillating Dipole)

With reference to Fig. 1.6.1 consider that a time varying current *I* is flowing in a very short and very thin wire of length *dl* in the z-direction. This current is given by *Idl cos ωt*. Since the current is in the z direction, the current density *J* will have only a z-component (i.e., J = $J_z a_z$). The vector magnetic potential A will also have only a z-component (i.e., A = $A_z a_z$).

Thus $\Delta^2 A = \Delta^2 A_Z = -\mu J_Z$ ...1.6.1

Though the cylindrical coordinate system can suitably accommodate the configuration of a filamentary current carrying conductor, wherein only the Az component exists and the Aρ and Aϕ components are zero.

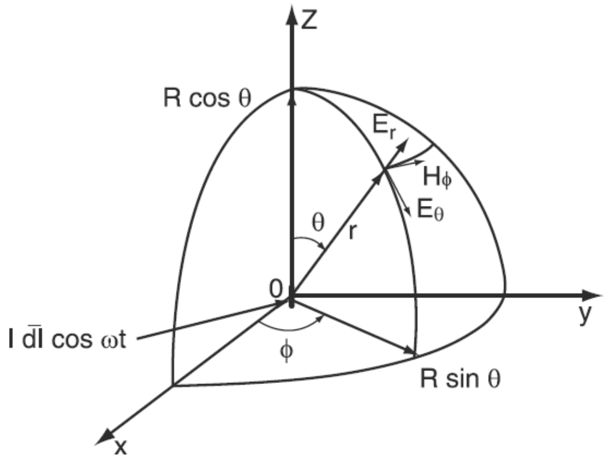


Figure 1.6.1 Configuration of filamentary current carrying conductor

But since the 3-dimensional radiation problem needs to be tackled in spherical coordinate system, Az is to be transformed to the spherical coordinate system. This transformation results in

$$A_r = A_Z Cos\theta, A_\theta = -A_Z Sin\theta and A_\phi = \theta \quad ...1.6.2$$

In view of the relation $Id\bar{l} = \bar{K}ds = \bar{J}dv$ for filamentary current, eq 1.5.25of sec. 1.4 can be written as

$$A_Z = \frac{\mu}{4\pi}\frac{Idlcos\omega(t - r/v)}{r} \quad ...1.6.3$$

In view of 1.6.2 and 1.6.3,

$$A_r = \frac{\mu}{4\pi}\frac{Idlcos\omega(t - r/v)}{r}cos\theta \; and \; A_\theta = -\frac{\mu}{4\pi}\frac{Idlcos\omega(t - r/v)}{r}sin\theta \quad ...1.6.4$$

Further from the relation B = ∇×A, the components of∇×A are obtained as below,

$$(\nabla \times A)_r = \frac{1}{sin\,\theta}\left[\frac{\partial}{\partial\theta}(sin\,\theta\, A_\Phi - \frac{\partial A_\theta}{\partial\Phi})\right] = B_r = 0 \quad ...1.6.5(a)$$

$$(\nabla \times A)_\theta = \left[\frac{1}{r\,sin\,\theta}\frac{\partial A_r}{\partial\Phi}(sin\,\theta\, A_\Phi - \frac{1}{r}\frac{\partial(rA_\Phi)}{\partial r})\right] = B_\theta = 0 \quad ...1.6.5(b)$$

$$(\nabla \times A)_\Phi = \frac{1}{r}\left[\frac{\partial}{\partial r}(rA_\theta - \frac{\partial A_r}{\partial\theta})\right] = B_\Phi = \mu H_\Phi \quad ...1.6.5(c)$$

From eq 1.6.5 it may be noted that only H_Φ survives. It can also be stated that ϕ derivative is zero (i.e., ∂/∂ϕ ≡ 0) for all field components due to the symmetry along ϕ. From equations 1.6.2 &1.6.5c,

$$H_\Phi = \frac{Idlsin\theta}{4\pi}\left[\frac{-\omega}{rv}sin\omega\left(t - \frac{r}{v}\right) + \frac{cos\omega(t - r/v)}{r^2}\right] \quad ...1.6.5$$

Now we know that,

$$E = \frac{1}{\varepsilon}\int(\nabla \times H)dt$$

Thus $E_r = \frac{1}{\varepsilon}\int(\nabla \times H)_r dt \; and \; E_\theta = \frac{1}{\varepsilon}\int(\nabla \times H)_\theta dt$ ...1.6.6

Since $\nabla \times H = \frac{1}{r\sin\theta}\frac{\partial(H_\phi \sin\theta)}{\partial\theta}a_r - \frac{1}{r}\frac{\partial(rH_\phi)}{\partial r}a_\theta$ its components in radial directions are

$$(\nabla \times H)_r = \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}\left[\frac{Idl}{4\pi}\sin^2\theta\left\{\frac{-\omega}{rv}sin\omega\left(t-\frac{r}{v}\right)+\frac{cos\omega(t-r/v)}{r^2}\right\}\right] = E_r \quad \text{...1.6.7}$$

From equations 1.6.6 and 1.6.7

$$E_r = \frac{Idl}{4\pi r}cos\theta\left[\frac{\omega}{rv}sin\omega\left(t-\frac{r}{v}\right)-\frac{cos\omega(t-r/v)}{r^2}\right] \quad \text{...1.6.8}$$

Putting *t'=t-r/v*

$$E_r = \frac{2Idlcos\theta}{4\pi\varepsilon}\left[\frac{\cos\omega t'}{r^2 v}+\frac{\sin\omega t'}{\omega r^3}\right] \quad \text{...1.6.9(a)}$$

Similarly,

$$E_\theta = \frac{Idl\sin\theta}{4\pi\varepsilon}\left[-\frac{\omega\sin\omega t'}{rv^2}+\frac{\cos\omega t'}{r^2 v}+\frac{\sin\omega t'}{\omega r^3}\right] \quad \text{...1.6.9(b)}$$

$$E_\theta = \frac{Idl\sin\theta}{4\pi\varepsilon}\left[-\frac{\omega\sin\omega t'}{rv^2}+\frac{\cos\omega t'}{r^2 v}+\frac{\sin\omega t'}{\omega r^3}\right]$$

$$E_\theta = \frac{Idl\sin\theta}{4\pi\varepsilon}\left[-\frac{\omega\sin\omega t'}{rv^2}+\frac{\cos\omega t'}{r^2 v}+\frac{\sin\omega t'}{\omega r^3}\right]$$

$$E_\theta = \frac{Idl\sin\theta}{4\pi\varepsilon}\left[-\frac{\omega\sin\omega t'}{rv^2}+\frac{\cos\omega t'}{r^2 v}+\frac{\sin\omega t'}{\omega r^3}\right]$$

H_φ can also be written as,

$$H_\phi = \frac{Idl\sin\theta}{4\pi}\left[\frac{\cos\omega t'}{r^2}-\frac{\omega\sin\omega t'}{rv}\right] \quad \text{...1.6.10}$$

It can be noted that the magnitudes of the two bracketed terms will become equal if the following relation is satisfied:

$$\frac{1}{\mathrm{r}} = \frac{\omega}{\mathrm{rv}} \text{ or } \mathrm{r} = \frac{v}{\omega} = \frac{f\lambda}{2\pi f} = \frac{\lambda}{2\pi}\ or\ r \approx \frac{\lambda}{6} \quad \text{...1.6.11}$$

From 1.6.11 it can be concluded that for $r < \lambda/6$, the induction field will dominate whereas for $r > \lambda/6$, the radiation field assumes more importance. Thus for $r >> \lambda/6$, only the radiation field needs to be accounted.

The expressions of E_θ, E_r and H_φ given by 1.6.9 and 1.6.10 involve three types of terms, which represent three different types of fields. These are noted below:

1. The terms inversely proportional to r^3 represent electrostatic field. Such terms are involved in the expressions of E_θ and E_r .
2. The terms inversely proportional to r^2 represent induction or near field. Such terms are involved in all the field components, i.e., in E_θ , E_r and H_φ.
3. Lastly, the terms which are inversely proportional only to r represent radiation (distant or far) field and are involved in the expressions of E_θ and H_φ.

1.7 The Hertzian Dipole - Relation between a Current Element and Electric Dipole

Ques: Write a note on Hertzian Dipole.

What is Hertzian Dipole? Write the relation between a current element and an electric dipole writing suitable expressions.

A Hertzian Dipole is nothing but an infinitesimal current element I dL. Actually such a current element does not exist in real life, but is serves as a building block in calculating the field of a practical antenna using integration. It is observed that electric field of the alternating current element contains the terms which corresponds to the field of an electric dipole.

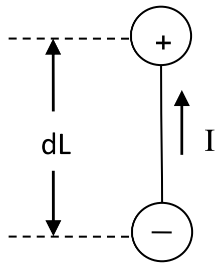


Figure 1.7.1 Hertzian Dipole

A Hertzian dipole consisting two equal and opposite charges at the end of the current element separated by a short distance dL as shown in figure 1.7.1. The wires between the two spheres where charges can accumulate is very thin as compared to the radius of spheres. Thus the current I is uniform through the wires. Also the distance dL is greater as compared to the redii of the spheres.
Let the current through the wires is sinusoidal,

$$\mathrm{i} = \mathrm{I}\cos\omega\mathrm{t} \qquad \text{...1.7.1}$$

Then the charge accumulated at the ends of the element and current flowing through the wires are related to each other by the expression,

$$\mathrm{i} = \frac{dq}{dt} = \mathrm{I}\cos\omega\mathrm{t} \qquad \text{...1.7.2}$$

therefore $\mathrm{dq} = \mathrm{I}\cos\omega\mathrm{t}\,\mathrm{dt}$ ... Separating variables

Integrating both sides with respect to corresponding variables, we get

$$q = \frac{\mathrm{I}\sin\omega\mathrm{t}}{\omega} \qquad \text{...1.7.3}$$

The expressions for the electric field components due to the separate charges at the ends of the current element are given by

$$E_r = \frac{2\,\mathrm{q}\,\mathrm{dL}\cos\theta}{4\pi\varepsilon\mathrm{r}^3} \qquad \text{...1.7.4}$$

and $$E_\theta = \frac{\mathrm{q}\,\mathrm{dL}\sin\theta}{4\pi\varepsilon\mathrm{r}^3} \qquad \text{...1.7.5}$$

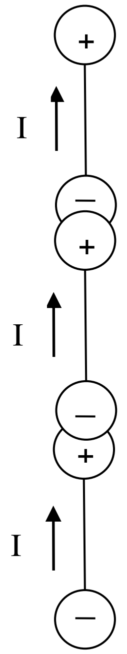
Substituting the values of q, in terms of current I from equations 1.7.3 to 1.7.4 and 1.7.5, the expression for the electric field components are given by

$$E_r = \frac{2\,\mathrm{I}\,\mathrm{dL}\cos\theta\sin\omega\mathrm{t}'}{4\pi\varepsilon\omega\mathrm{r}^3} \qquad \text{...1.7.6}$$

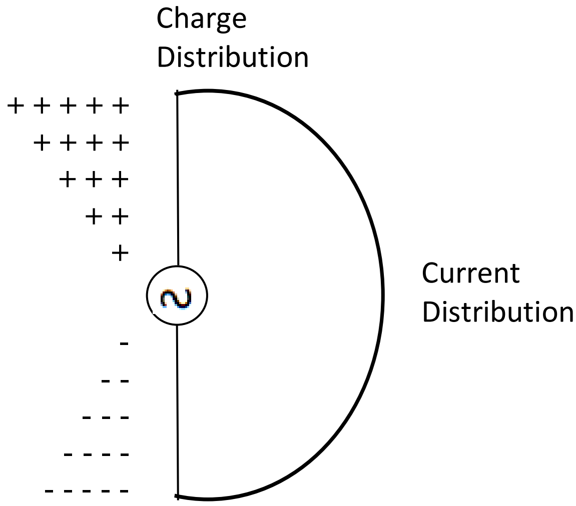
and $$E_\theta = \frac{\mathrm{I}\,\mathrm{dL}\sin\theta\sin\omega\mathrm{t}'}{4\pi\varepsilon\omega\mathrm{r}^3} \qquad \text{...1.7.7}$$

From the above two equations it is clear that these are the terms which appear in the expressions for the electric field due to the current element.

When such Hertzian Dipoles are connected end to end forming a practical antenna, it is observed that the positive charge at one end of the dipole gets cancelled by the equal and opposite charge at lower end of the next dipole. Hence when the current is uniform along the antenna, then there is no charge accumulation at the ends of dipole which indicates that $1/r^3$ term is absent and only induction and radiation fields are present. The chain of Hertzian dipole forming part of the antenna is shown in figure 1.7.2 (a).



(a) Chain of Hertzian Dipole



(b) Charge and current distribution of linear antenna

Figure 1.7.2

But if the current through the antenna is not uniform throughout then there is an accumulation of charge as shown in figure 1.7.2 (b), these charges cause stronger electric field component normal to the surface of the wire.

1.8 Power radiated by a Current Element

Consider a current element placed at a centre of a spherical co-ordinate system. Then the power radiated per unit area at a point P can be calculated by using Poynting Theorem. The power flow per unit area is given by Poynting Vector.
According to Poynting Theorem, the instantaneous power is given by,

$$\overline{P} = \overline{E} \times \overline{H} \qquad \text{...1.8.1}$$

The Components of the Poynting Vector are given by,

$$P_r = E_\theta H_\phi$$

$$P_\theta = -E_r H_\phi \qquad \text{...1.8.2}$$

And $$P_\phi = E_\phi H_r$$

But we know that when current element is placed at the origin, then the E_ϕ component of the electric field is zero. In other words, the Poynting vector will have only θ and r components.
Let us rewrite the field components of the electric and magnetic fields due to the current element, replacing v by c for the propagation in free space,

$$E_r = \frac{2Idl cos\theta}{4\pi\varepsilon}\left[\frac{\cos\omega t'}{cr^2} + \frac{\sin\omega t'}{\omega r^3}\right]$$

$$E_\theta = \frac{Idl\sin\theta}{4\pi\varepsilon}\left[-\frac{\omega\sin\omega t'}{c^2 r} + \frac{\cos\omega t'}{cr^2} + \frac{\sin\omega t'}{\omega r^3}\right]$$

$$H_\phi = \frac{Idl\sin\theta}{4\pi}\left[\frac{\cos\omega t'}{r^2} - \frac{\omega\sin\omega t'}{rc}\right]$$

The θ component of the instantaneous Poynting vector is given by

$$P_\theta = -E_r H_\phi$$

$$= \frac{-2Idl\cos\theta}{4\pi\varepsilon}\left[\frac{\cos\omega t'}{cr^2}+\frac{\sin\omega t'}{\omega r^3}\right]\frac{Idl\sin\theta}{4\pi}\left[\frac{-\omega\sin\omega t'}{cr}+\frac{\cos\omega t'}{r^2}\right]$$

$$= \frac{-2I^2dl^2\sin\theta\cos\theta}{16\pi^2\varepsilon}\left[\frac{-\omega\sin\omega t'\cos\omega t'}{c^2r^3}+\frac{\cos^2\omega t'}{cr^4}-\frac{-\omega\sin^2\omega t'}{\omega cr^4}+\frac{\sin\omega t'\cos\omega t'}{\omega r^5}\right]$$

Using property 2sinθcosθ=sin2θ

$$= \frac{I^2dl^2\sin 2\theta}{16\pi^2\varepsilon}\left[\frac{\omega\sin 2\omega t'}{2c^2r^3}-\frac{\cos^2\omega t'}{cr^4}+\frac{\omega\sin^2\omega t'}{\omega cr^4}-\frac{\sin 2\omega t'}{2\omega r^5}\right]$$

$$= \frac{I^2dl^2\sin 2\theta}{16\pi^2\varepsilon}\left[\frac{\omega\sin 2\omega t'}{2c^2r^3}+\frac{1}{cr^4}\left(\sin^2\omega t'-\cos^2\omega t'\right)-\frac{\sin 2\omega t'}{2\omega r^5}\right]$$

Now $\frac{1}{cr^4}\left[\sin^2\omega t'-\cos^2\omega t'\right]=\frac{1}{cr^4}\left[\frac{1-\cos 2\omega t'}{2}-\left(\frac{1+\cos 2\omega t'}{2}\right)\right]$

$$= \frac{1}{cr^4}\left[\frac{-2\cos 2\omega t'}{2}\right]=\frac{-\cos 2\omega t'}{cr^4}$$

Substituting value of the term in the original expression,

$$P_\theta = \frac{I^2dl^2\sin 2\theta}{16\pi^2\varepsilon}\left[\frac{\omega\sin 2\omega t'}{2c^2r^3}-\frac{-\cos 2\omega t'}{cr^4}-\frac{\sin 2\omega t'}{2\omega r^5}\right] \quad \text{...1.8.3}$$

The average values of sin2ωt' and cos2ωt' terms over a complete cycle is zero. This clearly indicates that for any value of r, the average value of P_θ is always zero over a complete cycle. Thus there will be the power flow back and forth in θ-direction only. Hence in θ-direction, there will be no net or average flow of power.

Let us calculate now radial component of the Poynting vector,

$$P_r = -E_\theta H_\phi$$

$$= \frac{Idl\sin\theta}{4\pi\varepsilon}\left[\frac{-\omega\sin\omega t'}{c^2r}+\frac{\cos\omega t'}{cr^2}+\frac{\sin\omega t'}{\omega r^3}\right]\frac{Idl\sin\theta}{4\pi}\left[\frac{-\omega\sin\omega t'}{cr}+\frac{\cos\omega t'}{r^2}\right]$$

$$= \frac{I^2dl^2\sin^2\theta}{16\pi^2\varepsilon}\left[\frac{\omega^2\sin^2\omega t'}{c^3r^2}-\frac{\omega\sin\omega t'\cos\omega t'}{c^2r^3}-\frac{\omega\sin\omega t'\cos\omega t'}{c^2r^3}+\frac{\cos^2\omega t'}{cr^4}-\frac{-\omega\sin^2\omega t'}{\omega cr^4}+\frac{\sin\omega t'\cos\omega t'}{\omega r^5}\right]$$

$$= \frac{I^2dl^2\sin^2\theta}{16\pi^2\varepsilon}\left[\frac{\omega^2\sin^2\omega t'}{c^3r^2}-\frac{\omega\sin 2\omega t'}{2c^2r^3}-\frac{\omega\sin 2\omega t'}{2c^2r^3}+\frac{\cos^2\omega t'}{cr^4}-\frac{-\omega\sin^2\omega t'}{\omega cr^4}+\frac{\sin 2\omega t'}{2\omega r^5}\right]$$

$$= \frac{I^2dl^2\sin^2\theta}{16\pi^2\varepsilon}\left[\frac{\omega^2}{c^3r^2}\left(\frac{1-\cos 2\omega t'}{2}\right)-\frac{\omega\sin 2\omega t'}{2c^2r^3}-\frac{\omega\sin 2\omega t'}{2c^2r^3}+\left(\frac{1+\cos 2\omega t'}{2cr^4}\right)-\left(\frac{1-\cos 2\omega t'}{2cr^4}\right)+\frac{\sin 2\omega t'}{2\omega r^5}\right]$$

$$= \frac{I^2dl^2\sin^2\theta}{16\pi^2\varepsilon}\left[\frac{\omega^2\left(1-\cos 2\omega t'\right)}{2c^3r^2}-\frac{\omega\sin 2\omega t'}{c^2r^3}+\frac{\cos 2\omega t'}{cr^4}+\frac{\sin 2\omega t'}{2\omega r^5}\right]$$

Rearranging the terms,

$$P_r = \left[\frac{I^2dl^2\sin^2\theta}{16\pi^2\varepsilon}\right]\left[\frac{\sin 2\omega t'}{2\omega r^5}+\frac{\cos 2\omega t'}{cr^4}-\frac{\omega\sin 2\omega t'}{c^2r^3}+\frac{\omega^2\left(1-\cos 2\omega t'\right)}{2c^3r^2}\right] \quad \text{...1.8.4}$$

Again the average values of sin2ωt' and cos2ωt' terms over a complete cycle is zero. Hence the average radial power is given by

$$P_r = \left[\frac{I^2 dl^2 \sin^2\theta}{16\pi^2\varepsilon}\right]\left[\frac{\omega^2}{2c^3r^2}\right]$$

$$\therefore P_r = \frac{\omega^2 I^2 dl^2 \sin^2\theta}{32\pi^2 r^2 c^3 \varepsilon}$$

$$\therefore P_r = \frac{1}{2\varepsilon c}\left(\frac{\omega I dl \sin\theta}{4\pi r c}\right)^2$$

But for free space, intrinsic impedance $\eta_0 = 1/\varepsilon c$

$$\therefore P_r = \frac{\eta_0}{2}\left(\frac{\omega I dl \sin\theta}{4\pi r c}\right)^2 \qquad \text{...1.8.5}$$

The power component represented by equation 1.83 is in radial direction. Hence it is called radial power. Equation 1.85 represents the average power flow.

The radiation terms in the expressions of the field contribute to this average power flow. When the point is away from the current element at far distance, the radiation terms contributes to the average power. But when the point is very close to the current element, the terms related to the induction and electrostatic fields are dominant and only 1/r terms contribute to the average power flow.

From the expressions of E_θ and H_ϕ, the amplitudes of the radiation fields only can be obtained. The amplitude from E_θ component is given by,

$$E_\theta = \frac{\omega I dl \sin\theta}{4\pi\varepsilon v^2 r}$$

$$\therefore E_\theta = \frac{(\omega / v) I dl \sin\theta}{(2\pi)(\varepsilon v) 2r}$$

But $\lambda = \frac{2\pi v}{\omega}\, and, \eta = \frac{1}{\varepsilon v}$

$$E_\theta = \frac{\eta I dl \sin\theta}{2\lambda r} \qquad \text{...1.8.6}$$

Similarly the amplitude from H_ϕ is given by,

$$H_\phi = \frac{\omega I dl \sin\theta}{4\pi v r}$$

$$\because H_\phi = \frac{I dl \sin\theta}{2\lambda r} \qquad \text{...1.8.7}$$

The radiation terms of E_θ and H_ϕ are in time phase and are related by

$$\frac{E_\theta}{H_\phi} = \eta \qquad \text{...1.8.8}$$

The total power radiated by the current element can be obtained by integrating the radial Poynting Vector over a spherical surface. Consider a spherical shell with the current element IdL placed at a centre of the spherical co-ordinate system as shown in figure 1.81. The point P at which power radiated is to be calculated is independent of an azimuthal angle ϕ, so the element of area ds on the spherical shell is considered as strip.

The element of area ds is given by

$$ds = 2\pi r^2 \sin\theta d\theta \quad \text{...1.8.9}$$

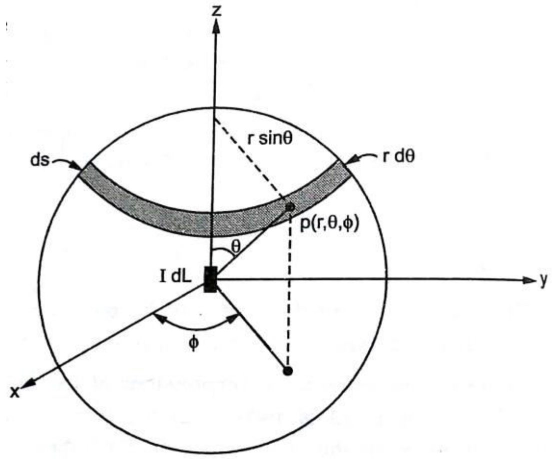


Figure 1.81 Element of area on spherical shell in the form of strip

The total power radiated is calculated by integrating average radial power over the spherical surface,

$$Power = \oiint_{surface} P_r ds \quad \text{...1.8.10}$$

$$= \oiint_{surface} \frac{\eta_0}{2}\left(\frac{\omega IdL\sin\theta}{4\pi rc}\right)^2 \left(2\pi r^2 \sin\theta d\theta\right)$$

$$= \oiint_{surface} \left(\frac{\eta_0}{2}\right)\left(\frac{\omega^2 I^2 dL^2 \sin^2\theta}{16\pi^2 r^2 c^2}\right)\left(2\pi r^2 \sin\theta\right) d\theta$$

$$= \oiint_{surface} \frac{\eta_0 \omega^2 I^2 dL^2}{16\pi c^2} \sin^3\theta d\theta$$

$$= \frac{\eta_0 \omega^2 I^2 dL^2}{16\pi c^2} \oiint_{surface} \sin^3\theta d\theta \quad \text{...1.8.11}$$

In spherical co-ordinate system, θ varies from 0 to π. Hence putting limits of integration as,

$$Power = \frac{\eta_0 \omega^2 I^2 dL^2}{16\pi c^2} \int_0^{\pi} \sin^3\theta d\theta$$

$$\therefore Power = \frac{\eta_0 \omega^2 I^2 dL^2}{8\pi c^2} \int_0^{\pi/2} \sin^3\theta d\theta \left[\because \int_0^{\pi} \sin^3\theta d\theta = 2\int_0^{\pi/2} \sin^3\theta d\theta\right]$$

Using the reduction formula for calculating the integral. By the reduction formula,

$$\int_0^{\pi/2} \sin^n x dx = \left[\frac{n-1}{n}\right]\left[\frac{\pi}{2}\right] \quad \text{if n is even}$$

$$\int_0^{\pi/2} \sin^n x dx = \left[\frac{n-1}{n}\right] \quad \text{if n is odd}$$

Here n is 3 i.e. hence we can write,

$$\int_0^{\pi/2}\sin^3\theta d\theta=\left[\frac{3-1}{3}\right]=\frac{2}{3}$$

Substituting this value in the equation of power, we get

$$Power=\frac{\eta_0\omega^2I^2dL^2}{8\pi c^2}\left(\frac{2}{3}\right)$$

$$\therefore \boxed{Power=\frac{\eta_0\omega^2I^2dL^2}{12\pi c^2}} \quad \text{...1.8.12}$$

The power represented by the above equation is in terms of the maximum or peak current. We know that,

$$I_{eff}=\frac{I_m}{\sqrt{2}}$$

or $$I_m=\sqrt{2}I_{eff}$$

Thus the power can be expressed in terms of the effective current as

$$Power=\frac{\eta_0\omega^2\left(\sqrt{2}I_{eff}\right)^2dL^2}{12\pi c^2}$$

$$\therefore \boxed{Power=\frac{\eta_0\omega^2I_{eff}^2dL^2}{6\pi c^2}} \quad \text{...1.8.13}$$

For free space, η_0=120 π

and $$\frac{\omega}{c}=\frac{2\pi}{\lambda}\, i.e. \frac{\omega^2}{c^2}=\frac{4\pi^2}{\lambda^2}$$

Substituting values in equation 1.8.13

$$Power=\frac{(120\pi)\left(\frac{4\pi^2}{\lambda^2}\right)I_{eff}^2dL^2}{6\pi}$$

$$\therefore Power=\frac{80\pi^2I_{eff}^2dL^2}{\lambda^2}$$

$$\therefore \boxed{Power=80\pi^2\left(\frac{dL}{\lambda}\right)^2I_{eff}^2} \quad \text{...1.8.14}$$

Equation (1.8.14) is of the radiated power in terms of effective current. We know that power is in the form of I^2R. Thus the coefficient in the above equation is nothing but the resistance. This resistance is called the radiation resistance of the current element, and represented by R_{rad}.

and $$\therefore \boxed{R_{rad}=80\pi^2\left(\frac{dL}{\lambda}\right)^2}$$

1.9 Power radiated by a Current Element

The practical example of the centre-fed antenna is an elementary dipole. The length of such centre-fed antenna is very short in wavelength. The current amplitude of such antenna is maximum at the centre and it decreases uniformly to zero at ends. The current distribution of short dipole is as shown in figure 1.9.1.

If we consider same current I flowing through the hypothetical current element and the practical short dipole, both of same length, then the practical short dipole radiates only one-quarter of the power that is radiated by the current element. This is because the field strength at every point on the short dipole reduces to half of the values for the current element and the power density reduces to one quarter. So obviously for the same current, the radiation resistance for the short dipole is ¼ times of the current element.

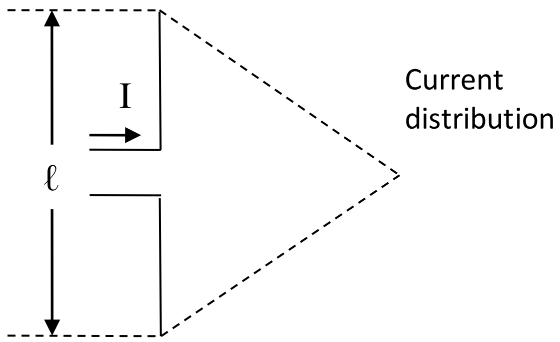


Figure 1.9.1 Current Distribution of Short Dipole

Hence the radiation resistance of the short dipole is given by

$$R_{rad}\left(short\_dipole\right)=\frac{1}{4}\left[80\pi^2\left(\frac{1}{\lambda}\right)^2\right]=20\pi^2\left(\frac{L}{\lambda}\right)^2$$

$$\therefore \boxed{R_{rad}\left(short\_dipole\right)\approx 200\left(\frac{L}{\lambda}\right)^2}$$

...1.9.1

Another practical example of an antenna is a monopole or short vertical antenna mounted on a reflecting plane as shown in figure 1.9.2.

Let the monopole is of length h. Again if we consider the same current I, flows through a monopole of length h and a short dipole of length l=2h then the field strength produced by both the antennas is same above the reflecting plane. But the monopole radiates only through the hemispherical surface above the plane. So the radiated power of a monopole is half of that radiated by a short dipole. Hence the radiation resistance of a monopole is half of the radiation resistance of the short dipole.

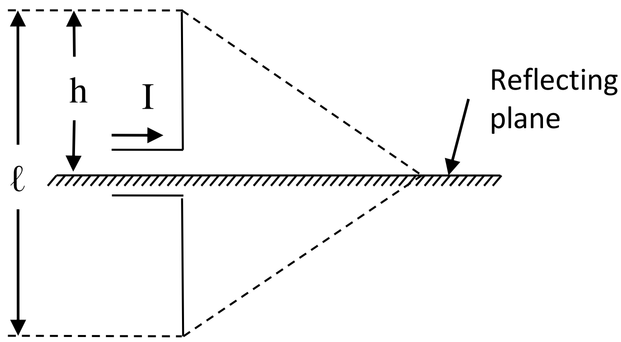


Figure 1.9.2 Current Distribution monopole

$$\therefore R_{rad}\left(monopole\right)=\frac{1}{2}\left[R_{rad}\left(short\_dipole\right)\right]=\frac{1}{2}\left[20\pi^2\left(\frac{L}{\lambda}\right)^2\right]$$

$$=10\pi^2\left(\frac{1}{\lambda}\right)^2$$

...1.9.2

But h= ℓ/2 for monopole

$$\therefore \quad R_{rad}\left(monopole\right)=10\pi^2\left(\frac{2h}{\lambda}\right)^2=40\pi^2\left(\frac{h}{\lambda}\right)^2$$

$$\therefore \quad \boxed{R_{rad}\left(monoppole\right)\approx 400\left(\frac{h}{\lambda}\right)^2}$$

...1.9.3

The expressions for the radiation resistance are valid only for the short antennas. But for the dipoles of length up to λ/4 wavelengths and the monopoles of heights up to λ/8 wavelengths, we can use these formulae directly.

1.10 Power radiated by the Half Wave Dipole and the Monopole

A Dipole antenna is a vertical radiator fed in the centre. It produces maximum radiation in the plane normal to the axis. For such a dipole antenna, the length specified is the overall length.
The vertical antenna of height H=L/2, produces the radiation characteristics above the plane which is similar to that produced by the dipole antenna of length L=2H. The vertical antenna is referred to as a monopole.
In general antenna requires large amount of current to radiate large amount of power. To generate such a large current at radio frequency is practically impossible. In case of Hertzian dipole the expression for $\bar{E}$ and $\bar{H}$ are derived assuming uniform current throughout the length. But we have studied that at the ends of the antenna current is zero. In other words the current is not uniform throughout the length as it is maximum at centre and zero at the ends. Hence practically Hertzian Dipole is not used. The practically used antennas are half wave dipole (λ/2) and quarter wave monopole (λ/4).
The half wave dipole consists of two legs each of length L/2. The physical length of the half wave dipole at the frequency of operation is (λ/2) in the free space.
The quarter wave monopole consists of a single vertical leg erected on the perfect ground i.e. perfect conductor. The length of the leg of the quarter wave monopole is (λ/4).
For the calculation of electromagnetic fields, the assumed sinusoidal current distributions along the half wave dipole and quarter wave monopole are as shown in figure 1.10.1 (a) and (b) respectively.

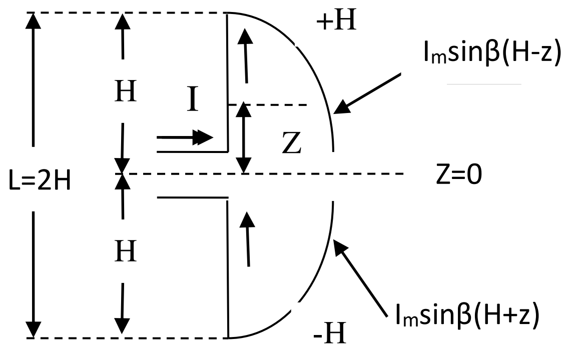


Figure 1.10.1 (a) Assumed sinusoidal current distribution in half wave dipole

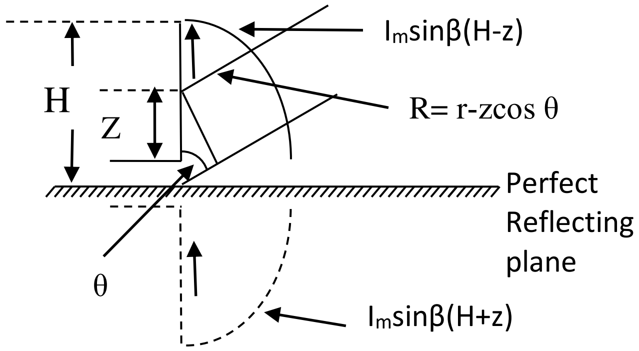


Figure 1.10.1 (b) Assumed sinusoidal current distribution in quarter wave monopole

Consider the assumed sinusoidal current distribution in the quarter wave monopole and half wave dipole. The current element Idz is placed at a distance z from z=0 plane. Let I_M be the maximum value of the current in the current element.
For a half wave dipole antenna, the total radiated power is given by

$$W = 80\pi^2 \left(\frac{dL}{\lambda}\right)^2 I_{eff}^2$$

The effective length L_{eff}=dL=2l/ π for sinusoidal current, therefore

$$W = 80\pi^2 \left(\frac{2l}{\pi\lambda}\right)^2 I_{eff}^2$$

For a half wave dipole the physical length l=λ/2, then

$$W = 80\pi^2 \left[\frac{2}{\pi}\frac{\lambda}{2\lambda}\right]^2 I_{eff}^2$$

$$\therefore \qquad W = 80 I_{eff}^2 Watts$$

and $\quad R_r = 80\Omega$

In actual the value of radiation resistance is around 73 Ohms.

For a **Monopole antenna**, the total radiated power will be equal to that of radiated by a short dipole i.e.

$$W = 10\pi^2 \left(\frac{dL}{\lambda}\right)^2 I_{eff}^2$$

The effective length L_{eff}=dL=2h for monopole antenna, therefore

$$W = 80\pi^2 \left(\frac{2l}{\pi\lambda}\right)^2 I_{eff}^2$$

$$W = 40\pi^2 \left[\frac{h}{\lambda}\right]^2 I_{eff}^2$$

$$\boxed{W \approx 400\left(\frac{h}{\lambda}\right)^2 I_{eff}^2} Watts$$

And the radiation resistance is given by

$$\boxed{R_r \approx 400\left(\frac{h}{\lambda}\right)^2 \Omega}$$

1.11 Sine and Cosine Integral
Near field due to Sinusoidal Current Distribution

Consider a dipole with sinusoidal current distribution as shown in figure 1.11.1.
Let P be the point at which field is to be calculated. The element length dh is located at a distance h from origin. The lower and upper tips of the dipole are located at –H and +H respectively.
The distance between upper tip of dipole and point P is given by

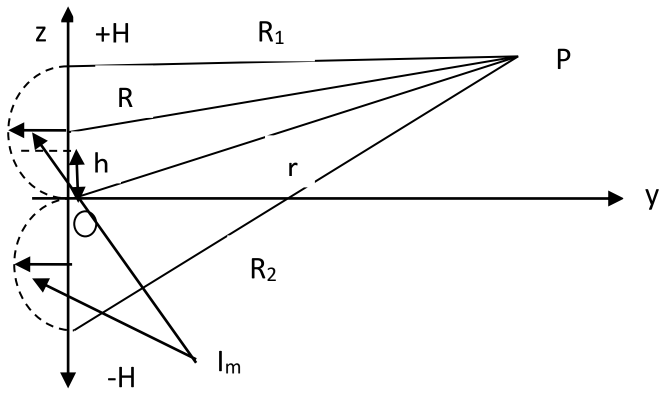


Figure 1.11.1. A Dipole with sinusoidal current distribution

$$R_1 = \sqrt{(z-H)^2 + y^2} \qquad ...1.11.1(a)$$

The distance between lower tip of dipole and point P is given by

$$R_1 = \sqrt{(z+H)^2 + y^2} \qquad ...1.11.1(b)$$

The distance between the element length dh and point P is given by

$$R_1 = \sqrt{(z-h)^2 + y^2} \qquad ...1.11.1(c)$$

The distance between origin and point P is given by

$$R_1 = \sqrt{z^2 + y^2} \qquad ...1.11.1(d)$$

The current distribution can be represented as

$$I = I_m \sin \beta(H-h), for, h>0.....(a)$$
$$I = I_m \sin \beta(H+h), for, h<0.....(b)$$ ...1.11.2

Using equations 1.11.1 and 1.11.2, the z component of the vector potential at point P is given by

$$A_z = \frac{\mu I_m}{j\beta\pi}\left[e^{j\beta H}\int_0^H \frac{e^{-j\beta(R+h)}}{R}dh - e^{-j\beta H}\int_0^H \frac{e^{-j\beta(R-h)}}{R}dh + e^{j\beta H}\int_{-H}^0 \frac{e^{-j\beta(R-h)}}{R}dh - e^{-j\beta H}\int_{-H}^0 \frac{e^{-j\beta(R+h)}}{R}dh \right]$$ ...1.11.3

Now $$B_\phi = \mu H_\phi = \left(\nabla \times A\right)_\phi = \frac{-\partial Az}{\partial \rho} = \frac{-\partial Az}{\partial y} = -\mu H_X$$ ...1.11.4

$$H_\phi = \frac{-I_m}{j8\pi}\left[e^{j\beta H}\int_0^H \frac{\partial}{\partial y}\left\{\frac{e^{-j\beta(R+h)}}{R}dh\right\} - e^{j\beta H}\int_0^H \frac{\partial}{\partial y}\left\{\frac{e^{-j\beta(R-h)}}{R}dh\right\} + e^{j\beta H}\int_{-H}^0 \frac{\partial}{\partial y}\left\{\frac{e^{-j\beta(R-h)}}{R}dh\right\} - e^{j\beta H}\int_{-H}^0 \frac{\partial}{\partial y}\left\{\frac{e^{-j\beta(R+h)}}{R}dh\right\} \right]$$

...1.11.5

Consider first integral term in equation 1.11.5

$$\int_0^H \frac{\partial}{\partial y}\left\{\frac{e^{-j\beta(R+h)}}{R}dh\right\} = \int_0^H \left[\frac{e^{-j\beta y}e^{-j\beta(R+h)}}{R^2} - \frac{ye^{-j\beta(R+h)}}{R^3}\right]dh$$ ...1.11.6

Integrating the above equation

$$= e^{j\beta H}\left[\frac{ye^{-j\beta(R+h)}}{R(R+h-z)}\right]_{h=0}^{h=H}$$

$$= e^{j\beta H}\left[\frac{e^{-j\beta(R_1+H)}}{R_1(R_1+H-z)} - \frac{(r+z)e^{-j\beta r}}{r(r-z)}\right]$$ ...1.11.7

But $$R_1^2-(H-z)^2 = r^2-z^2 = y^2$$

Therefore,

$$\int_0^H \frac{\partial}{\partial y}\left\{\frac{e^{-j\beta(R+h)}}{R}dh\right\} = \frac{e^{j\beta H}}{y}\left[\left(1-\frac{H-z}{R_1}\right)e^{-j\beta(R_1+H)} - \left(1+\frac{z}{r}\right)e^{-j\beta r}\right]$$

Similarly for other three integrals

$$\int_0^H \frac{\partial}{\partial y}\left\{\frac{e^{-j\beta(R-h)}}{R}dh\right\} = \frac{e^{j\beta H}}{y}\left[\left(1+\frac{H-z}{R_1}\right)e^{-j\beta(R_1-H)} - \left(1-\frac{z}{r}\right)e^{-j\beta r}\right]$$

$$\int_{-H}^0 \frac{\partial}{\partial y}\left\{\frac{e^{-j\beta(R-h)}}{R}dh\right\} = \frac{e^{-j\beta H}}{y}\left[\left(1-\frac{H+z}{R_2}\right)e^{-j\beta(R_2+H)} - \left(1-\frac{z}{r}\right)e^{-j\beta r}\right]$$

$$\int_{-H}^0 \frac{\partial}{\partial y}\left\{\frac{e^{-j\beta(R+h)}}{R}dh\right\} = \frac{e^{-j\beta H}}{y}\left[\left(1+\frac{H+z}{R_2}\right)e^{-j\beta(R_2-H)} - \left(1-\frac{z}{r}\right)e^{-j\beta r}\right]$$

Therefore the overall magnetic field intensity is given by,

$$H_\phi = \frac{-\mathrm{I_m}}{4j\pi}\left[\frac{e^{-j\beta R_1}}{y} + \frac{e^{-j\beta R2}}{y} - \frac{2\cos\beta H e^{-j\beta r}}{y}\right]$$ ...1.11.8

We know the Maxwell's equations,

$$\nabla\times\overline{H}=\frac{\partial\overline{D}}{\partial t}=j\omega\varepsilon\overline{E} \quad \text{or} \quad \overline{E}=\frac{1}{j\omega\varepsilon}\left(\nabla\times\overline{H}\right) \quad \text{In free space.}$$

$$\therefore \quad E_y=-\frac{1}{j\omega\varepsilon}\frac{\partial H_\phi}{\partial z} \quad \text{or} \quad E_z=-\frac{1}{j\omega\varepsilon y}\frac{\partial\left(yH_\phi\right)}{\partial y} \qquad \text{...1.11.9}$$

But for $\overline{H}$, H_ϕ is the only existing component, hence

$$\nabla\times\overline{H}=\frac{\partial H_\phi}{\partial z}\overline{a}_y+\frac{1}{y}\frac{\partial(yH_\phi)}{\partial y}\overline{a}_z \qquad \text{...1.11.10}$$

Therefore by equation 1.11.9 and 1.11.10, we can write

$$E_z=\frac{-j\beta \mathrm{I_m}}{4\pi\varepsilon\omega y}\left[\frac{ye^{-j\beta R_1}}{R_1}+\frac{ye^{-j\beta R_2}}{R_2}-2\cos\beta H\frac{ye^{-j\beta r}}{r}\right]$$

$$E_z=-j30\mathrm{I_m}\left[\frac{e^{-j\beta R_1}}{R_1}+\frac{e^{-j\beta R_2}}{R_2}-2\cos\beta H\frac{ye^{-j\beta r}}{r}\right] \qquad \text{...1.11.11}$$

$$E_y=j30\mathrm{I_m}\left[\frac{z-H}{y}\frac{e^{-j\beta R_1}}{R_1}+\frac{z+H}{y}\frac{e^{-j\beta R_2}}{R_2}-2\frac{z\cos\beta H}{y}\frac{e^{-j\beta r}}{r}\right] \qquad \text{...1.11.12}$$

$$H_\phi=\frac{j30\mathrm{I_m}}{\eta y}\left[e^{-j\beta R_1}+e^{-j\beta R_2}-2\cos\beta He^{-j\beta r}\right] \qquad \text{...1.11.13}$$

From the above equations we can write that,

1. The term $e^{-j\beta R_1}$ indicates that spherical wave is originating at the top of the antenna.
2. Similarly $e^{-j\beta R_2}$ indicates that spherical wave is originating from
   a) Bottom of the antenna if it is a dipole or
   b) Lower tip of the antenna if it is a monopole.

3. The term $e^{-j\beta r}$ indicates that spherical wave is originating from the centre of the antenna for dipole and at the base if monopole.
4. The amplitude of the wave from centre of antenna depends on the length of the antenna(H), for example for a half wave dipole or quarter wave monopole H=λ/4, the amplitude becomes zero because

$$\cos\beta H=\cos\left(\frac{2\pi}{\lambda}\right)\left(\frac{\lambda}{4}\right)=\cos\left(\frac{\pi}{2}\right)=0$$

1.12 Near field(Induction Field) and Far Field(Radiation Field)

We know the magnetic field component is given by

$$H_\phi=\frac{Idl\sin\theta}{4\pi}\left[+\frac{\cos\omega t'}{r^2}-\frac{\omega\sin\omega t'}{rc}\right] \qquad \text{...1.12.1}$$

Amplitude I Term II Term

(Induction Field) (Radiation Field)

It consist of two terms

(i) Induction (Near) Field: The first term varies inversely as square of the distance (i.e. $1/r^2$) and is known as near field or induction field. It predominates at points near to the current element where r is small. The field is more effective near the current element only. It represents the energy stored in the magnetic field surrounding the conductor. This energy is alternatively stores in the field and returned to the source alternatively during each half cycle. The induction field is not so important from the radiation point of view and therefore neglected.

(ii) Radiation (Far) Field: The second term varies inversely as distance (i.e. 1/r) and is known as the Radiation Field or Far Field or the Distant Field, which accounts for the radiation of the Electromagnetic waves from a conductor under the suitable conditions. This radiation field is of great importance at large distance.

The radiation component of the magnetic field is produced by the alternating electric field and the electric radiation components occur from the alternating magnetic field. The flow of current in the conductor creates the local induction fields, whereas the radiation fields exist as a consequence of induction fields.

Near the conductor the magnetic field is in phase with the current in the conductor, whereas the electric field varies in phase with the change on either end of the conductor element. In this region the Electric and magnetic fields have, a phase difference of $\pi/2$ radians and are at right angles to each other in space i.e. E_θ and H_φ are on phase in far field.

The radiation field E_θ at $r>>\lambda$ is given by (from section 1.8)

$$E_\theta = \frac{Idl\sin\theta}{4\pi\varepsilon}\left[-\frac{\omega\sin\omega t'}{c^2 r}+\frac{\cos\omega t'}{cr^2}+\frac{\sin\omega t'}{\omega r^3}\right]$$

$$E_\theta = \frac{Idl\sin\theta}{4\pi\varepsilon}\left[-\frac{\omega\sin\omega t'}{c^2 r}\right]$$ ($1/r^2$ and $1/r^3$ terms are neglected)

$$E_\theta = -\frac{\omega Idl\sin\theta\sin\omega t'}{4\pi\varepsilon c.cr} = -\frac{2\pi fIdl\sin\theta\sin\omega t'}{4\pi\varepsilon\frac{1}{\sqrt{\mu\varepsilon}}cr} \qquad c=\frac{1}{\sqrt{\mu\varepsilon}}$$

$$E_\theta = -\frac{Idl\sin\theta\sin\omega t'}{2\sqrt{\frac{\varepsilon}{\mu}}\lambda r} = -\frac{\eta Idl\sin\theta\sin\omega t'}{2\lambda r} \qquad \eta=\sqrt{\frac{\mu}{\varepsilon}}=120\pi$$

$$\boxed{E_\theta = -\frac{60\pi Idl\sin\theta\sin\omega\left(t-\frac{r}{c}\right)}{\lambda r}} \qquad ...1.12.2$$

$$\boxed{|E_\theta| = -\frac{60\pi Idl}{\lambda r}} \qquad ...1.12.3$$

Similarly at distance $r>>\lambda$

$$H_\phi = \frac{Idl\sin\theta}{4\pi}\left[\frac{\cos\omega t'}{r^2}-\frac{\omega\sin\omega t'}{rc}\right] \quad \text{Or} \quad H_\phi = \frac{Idl\sin\theta}{4\pi}\left[-\frac{\omega\sin\omega t'}{rc}\right] \qquad \frac{1}{r^2}\approx 0$$

$$H_\phi = -\frac{2\pi fIdl\sin\theta\sin\omega t'}{4\pi cr}$$

$$H_\phi = -\frac{Idl \sin\theta \sin\omega\left(t - \frac{r}{c}\right)}{2\lambda r}$$

$$|H_\phi| = \left|\frac{Idl \sin\theta \sin\omega t'}{2\lambda r}\right|$$ Maximum value, when θ=90⁰ ...1.12.4

$$H_\phi = \frac{Idl}{2\lambda r}$$ ...1.12.5

The equations 1.12.2 and 1.12.4 constitute the field present in the radiating wave from the current element *I dl cosωt.*